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Parametrized relativistic dynamical framework for neutrino oscillations

John R. Fanchi

Department of Engineering and TCU Energy Institute
Texas Christian University
Fort Worth, Texas, 76129 USA

E-mail:**j.r.fanchi@tcu.edu**

Abstract. Mass state transitions are a key feature of parametrized relativistic dynamics (PRD). PRD is a manifestly covariant quantum theory with invariant evolution parameter. The theory has been applied to neutrino flavor oscillations between two mass states. It is generalized to transitions between three mass states and applied to the survival of electron neutrinos. The analysis shows that significant differences exist between theoretical results of the conventional model and the PRD model.

1. Introduction

Experiments with solar neutrinos, atmospheric neutrinos, reactor neutrinos, and accelerator neutrinos have demonstrated that flavor mixing can occur between two or three neutrino flavors composed of up to three neutrino mass states [1,2]. Mass state transitions are a key feature of parametrized relativistic dynamics (PRD). Introductions to PRD are presented by Fanchi [3, 4], Pavsic [5], and Horwitz [6]. A review of the highlights of the history of neutrinos is presented to establish a framework for analysing neutrino oscillations in the context of PRD. We then extend the PRD formalism for two-state flavor mixing [7] to three-state flavor mixing. The formalism is illustrated by applying it to the neutrino transition $\nu_e \rightarrow \nu_\mu$ from an electron neutrino ν_e to a muon neutrino ν_μ in vacuum. In this application we assume the transition $\nu_e \rightarrow \nu_\tau$ from an electron neutrino ν_e to a tau neutrino ν_τ is negligible and that only two flavor states are involved. Calculation of electron neutrino survival probabilities using both the conventional analysis and PRD shows that there are significant differences between the two theories.

1.1 The Neutrino Hypothesis

We begin our review of some highlights of the history of neutrinos with the observation that particles can be emitted by radioactive elements. Wilhelm Roentgen [8] observed a new kind of radiation called x-rays. Roentgen's x-rays could pass through a human body and create a shadowy image of bone. Henri Becquerel learned about x-rays and began to look for a connection between x-rays and the phosphorescence of a uranium salt. Becquerel [9] published a series of articles about his discovery ofa

radiation that appeared without the stimulation of sunlight and could penetrate matter. The radiation discovered by Becquerel was not the same as Roentgen's x-rays and was later named radioactivity by Pierre and Marie Curie.

J.J. Thomson [10] recognized that "cathode rays" are negatively charged particles. Thomson used a magnet to alter the trajectory of the cathode rays and measure the mass-to-charge ratio. Cathode rays are now known as electrons. Robert Millikan [11, 12] used the oil drop experiment to measure the charge of the cathode ray. The mass of the electron was determined by combining Millikan's charge measurement with Thomson's mass-to-charge ratio.

Ernest Rutherford [13] was the first to observe that uranium emitted at least two distinct types of rays that he called alpha radiation and beta radiation. Shortly thereafter Paul Villard [14] studied the decay of radium and discovered a third type of electrically neutral radiation that Rutherford [15] later named gamma radiation. Today we know that alpha radiation is the helium nucleus, beta radiation is an electron, and gamma radiation is an energetic photon.

Rutherford [16] bombarded thin gold foils with alpha particles. The scattering of alpha particles by atoms in the foil showed that most of the atomic mass must occupy a small space inside the atom. This mass was called the nucleus. The constituents of the nucleus were later identified as the proton and a new, electrically neutral particle called the neutron. At this point in time, the atom was thought to contain a massive, positively charged nucleus surrounded by a distribution of negatively charged particles.

Experimental studies of radioactive elements that emitted alpha particles or gamma particles showed that the decays satisfied conservation of energy. By contrast, studies of radioactive elements that emitted beta particles did not appear to conserve energy. During beta decay, the positive charge of the nucleus increases by one positive charge in association with the emission of a negatively charged beta particle. James Chadwick [17] measured kinetic energy of emitted beta particles by passing the negatively charged beta particles through a magnetic field and calculating the kinetic energy from the deflection of the beta particle trajectory in the magnetic field. Chadwick showed that the kinetic energy of beta particles from beta decay experiments displayed a broad, continuous distribution of kinetic energy. The beta particles did not have enough kinetic energy to account for all of the energy needed to conserve energy in the beta decay process. Was energy conservation being violated?

Neils Bohr was a proponent of the idea that energy conservation was being violated [18, page 63 ff).Wolfgang Pauli was not prepared to abandon energy conservation. In a 1930 letter to a conference organizer, Pauli suggested "the possibility that inside the nuclei there are particles electrically neutral, that I will call neutrons, which have spin ½ and follow the exclusion principle" [19, pages 16-17]. Pauli's neutron was later named neutrino (Italian for "little neutral one") by Enrico Fermi [20] to distinguish it from the more massive neutron discovered by Chadwick [21].

The concept of a neutron was introduced by Rutherford in 1920. He suggested that "the nuclei of atoms contain electrons as well as positively charged bodies, and that the positive charge of the nucleus represents the excess positive charge." [22, page 377]. The basic building blocks were the positively charged nucleus of the hydrogen atom (the proton) and the negatively charged electron. In the case of a single electron and a single hydrogen nucleus, "it may be possible for an electron to combine much more closely with the H nucleus to form a kind of neutral doublet" [22, page 396]. The neutron was discovered in 1932 [21]. Was the neutron the bound state of an electron and a proton, or was it a new particle?

The beta decay process was evidence that the neutron was not a stable particle. Fermi [20] adopted Pauli's "neutron" concept, which he renamed "neutrino," and hypothesized a 4-point beta decay in which the neutron decayed into a proton, an electron, and a neutrino. The study of cosmic rays led Sakata and Inoue [23] to suggest that there may be two neutrinos: the electron neutrino ν_e associated with electrons, and the muon neutrino ν_μ associated with muons discovered in cosmic rays.

1.2 Detecting the Neutrino

It was reasonable to infer from Fermi's 4-point theory that the neutrino could interact with other particles in such a way that it could be detectable. One interaction is beta decay in which the neutron decays into a proton, an electron, and an electron antineutrino: $n^0 \to p^+ + e^- + \bar{\nu}_e$. Another possible interaction is inverse beta decay in which an electron antineutrino interacts with a proton to form a neutrino and a positron: $\bar{\nu}_e + p^+ \to n^0 + e^+$. The electron capture process occurs when an electron in a lower energy level of an atom is captured by a proton in the nucleus to produce a neutron and a neutrino: $e^- + p^+ \to n^0 + \nu_e$.

It was not clear at the time if the neutrino and the antineutrino behaved differently when interacting with matter. Two possible beta emission processes were positive beta emission $p^+ \to n^0 + e^+ + \nu_e$ in which a proton transformed into a neutron, a positron and a neutrino, and negative beta emission $n^0 \to p^+ + e^- + \bar{\nu}_e$ in which a neutron transformed into a proton, an electron and an antineutrino. More generally, positive beta emission for a nucleus X with mass number A and atomic number Z can be written as $^A_Z X \to\ ^A_{Z-1} Y + e^+ + \nu_e$ where the symbol Y represents the resulting nucleus. Positive beta emission may be viewed as the transformation of a proton inside the nucleus to a neutron: $p^+ \to n^0 + e^+ + \nu_e$. Similarly, negative beta emission can be written as $^A_Z X \to\ ^A_{Z+1} Y + e^- + \bar{\nu}_e$ which may be viewed as the transformation of a neutron inside the nucleus to a proton: $n^0 \to p^+ + e^- + \bar{\nu}_e$.

Bruno Pontecorvo [24] proposed detecting neutrinos by allowing the neutrino to interact with a proton or neutron of a nucleus heavier than hydrogen so that $Z > 1$. The interaction would change a proton into a neutron and yield an isotope of a new element. Neutrino detection would be achieved by starting with a large amount of a pure isotope and look for the formation of a neutrino-generated element as a function of time. One reaction proposed by Pontecorvo was the reaction in which a neutrino interacts with a proton in a chlorine nucleus to produce an electron and an argon nucleus: $\nu_e +\ ^{37}_{17}Cl \to e^- +\ ^{37}_{18}Ar$. Ray Davis performed the experiment using radiation emitted by different reactors and detectors containing carbon tetrachloride.

Nuclear reactors emit antineutrinos from negative beta decays of fission products. Pontecorvo's reaction $\nu_e +\ ^{37}_{17}Cl \to e^- +\ ^{37}_{18}Ar$ assumed a neutrino would interact with a proton. Davis was aware that his experiment was testing the relative performance of neutrinos and antineutrinos: "If neutrinos and antineutrinos areidentical in their interactions with nucleons one shouldbe able to observe the process upon carrying the experimentto the required sensitivity. However, if neutrinosand antineutrinos differ in their interactions withnucleons one would not expect to induce the reaction $^{37}_{17}Cl(\bar{\nu}_e, e^-)^{37}_{18}Ar$." [25, pg. 766] Assuming the experiment had adequate sensitivity, Davis argued that a null result using chlorine in the detector implied that the interaction between neutrinos and nucleons is not the same as the interaction between antineutrinos and nucleons. His early results were not conclusive [25,26].

The first tentative observation of neutrinos was made by Reines and Cowan in 1953 at the Hanford nuclear reactor [27] on the Columbia River in the state of Washington. Electron neutrinos from nuclear fission reactors have since been identified as electron antineutrinos. Reines and Cowan observed the inverse beta decay process in which an electron antineutrino interacted with a proton to from a neutrino and a positron $\bar{v}_e + p^+ \rightarrow n^0 + e^+$. The positron produced by the reaction interacted with an electron in a pair-annihilation process to yield two gamma rays in the reaction $e^+ + e^- \rightarrow 2\gamma$.

The Hanford work identified gamma rays from pair annihilation, but this was not considered sufficiently conclusive because the gamma ray signal could not be distinguished from cosmic ray signals. The experiment was moved to the Savannah River Nuclear Reactor facility near Augusta, Georgia where it could be shielded from cosmic rays. The experiment was also modified to include a neutron absorber (cadmium chloride) in the scintillation tank. The neutron produced in the inverse beta decay process was absorbed by a cadmium nucleus in the neutron capture process $n^0 + {}^{108}_{48}Cd \rightarrow {}^{109}_{48}Cd + \gamma$. Photomultiplier tubes in the scintillation tank should be able to detect gamma rays from the pair-annihilation process approximately 5 microseconds before gamma ray signals from the neutron capture process. The neutrino signature was the combination of the detection of gamma rays from pair-annihilation followed by the detection of gamma rays from the neutron capture process. Cowan, Reines and collaborators[28] reported the direct detection of the electron neutrino in 1956.

1.3 Neutrino Oscillations

Ray Davis continued to improve his experimental techniques. Together with his colleagues, Davis increased the volume of ${}^{37}_{17}Cl$ in his detector and moved the detector underground to the Homestake gold mine in South Dakota. His goal was to minimize extraneous signals from cosmic rays as he sought to measure neutrino production by fusion reactions in the interior of the sun. Davis [29] found that solar neutrino flux was much lower than predicted by John Bahcall [30]. Bahcall and colleagues [31, 32] refined their theoretical predictions, but additional measurements by Davis and colleagues [32] showed that observed solar neutrino flux wasstill lower than predicted. Where were the missing solar neutrinos?

Work with kaons had shown that kaons could oscillate between neutral kaon $K^0$and neutral anti-kaon $\overline{K^0}$flavor states [33]. The observable kaons were the long-lived kaon state (symmetric superposition $K_L^0 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0})$) and short-lived kaon state (anti-symmetric superposition $K_S^0 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0})$). By analogy, Pontecorvo suggested that the electron neutrino v_e, and the muon neutrino v_μ could oscillate between states [34,35]. Danby, et al. [36] showedthat there is experimental evidence that there are at least two different types of neutrinos.Maki, Nakagawa and Sakata [37] presented a theory of 2-neutrino mixing between the muon neutrino and electron neutrino.

A third type of neutrino was proposed by Perl, et al. [38]. They were the first to report evidence of the tau lepton and suggested that a third neutrino, the tau neutrino, may be associated with the tau lepton. Thetau neutrino was observed in 2000 by the DONUT(Direct Observation of Nu Tau) collaboration at Fermilab in the USA and Super-Kamiokande in Japan [39]. Super-Kamiokandebegan operation in 1996. The collaboration detected oscillations in atmospheric neutrinos by 1998 [40], which implies that neutrinos have mass.

Non-technical reviews of the history of neutrinos and additional references are provided by Solomey [19], Franklin [18], Close [41], and Jayawardhana [42].

2. Mass Basis and Flavor Basis

We are interested in developing a formalism within the context of PRD that can describe transitions between three neutrino flavor states $\{|v_\alpha\rangle; \alpha = e, \mu, \tau\}$ given the assumption that neutrinos are composed of up to three mass states $\{|v_j\rangle; j = 1, 2, 3\}$. The mass and flavor states can be written as 3-component column vectors:

$$|v_j\rangle = \begin{bmatrix} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{bmatrix} \tag{2.1}$$

and

$$|v_\alpha\rangle = \begin{bmatrix} |v_e\rangle \\ |v_\mu\rangle \\ |v_\tau\rangle \end{bmatrix} \tag{2.2}$$

The mass basis $\{|v_j\rangle; j = 1, 2, 3\}$ is related to the flavor basis $\{|v_\alpha\rangle; \alpha = e, \mu, \tau\}$ by a unitary transformation:

$$\begin{bmatrix} |v_e\rangle \\ |v_\mu\rangle \\ |v_\tau\rangle \end{bmatrix} = U \begin{bmatrix} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{bmatrix} \tag{2.3}$$

where U is the unitary matrix

$$U = \begin{bmatrix} u_{e1} & u_{e2} & u_{e3} \\ u_{\mu1} & u_{\mu2} & u_{\mu3} \\ u_{\tau1} & u_{\tau2} & u_{\tau3} \end{bmatrix} \tag{2.4}$$

satisfying

$$U^{-1} = \left(U^*\right)^T \tag{2.5}$$

The elements of the unitary matrix are

$$u_{\alpha j}^{-1} = u_{j\alpha}^* \; ; \; j = 1, 2, 3 \; and \; \alpha = e, \mu, \tau \tag{2.6}$$

The expanded form of the unitary transformation is

$$|v_e\rangle = u_{e1}|v_1\rangle + u_{e2}|v_2\rangle + u_{e3}|v_3\rangle$$
$$|v_\mu\rangle = u_{\mu1}|v_1\rangle + u_{\mu2}|v_2\rangle + u_{\mu3}|v_3\rangle \tag{2.7}$$
$$|v_\tau\rangle = u_{\tau1}|v_1\rangle + u_{\tau2}|v_2\rangle + u_{\tau3}|v_3\rangle$$

A mass basis state satisfies the temporal evolution equation

$$T|v_j\rangle = i\hbar\frac{\partial}{\partial s}|v_j\rangle$$
$$= T_j|v_j\rangle \tag{2.8}$$
$$= \hbar^2\frac{k_j^\mu k_{j\mu}}{2m_j}|v_j\rangle$$

where T_j is the eigenvalue of the temporal operator T, s is the scalar evolution parameter, k_j^μ is the energy-momentum of state j and m_j is the mass of state j. Equation (2.8) has the formal solution

$$\begin{bmatrix} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{bmatrix} = \begin{bmatrix} e^{-i\frac{T_1 s}{\hbar}} & 0 & 0 \\ 0 & e^{-i\frac{T_2 s}{\hbar}} & 0 \\ 0 & 0 & e^{-i\frac{T_3 s}{\hbar}} \end{bmatrix} \begin{bmatrix} |v_1(0)\rangle \\ |v_2(0)\rangle \\ |v_3(0)\rangle \end{bmatrix} \tag{2.9}$$

where $v_j(0)$ is mass state j at $s=0$.

3. Transitions between Flavor States: Electron Neutrino Disappearance

The formalism for three mass states presented in the previous section is illustrated by applying the formalism to the disappearance of electron neutrinos. We begin with a pure beam of electron neutrinos in flavor state $|v_e\rangle$. The probability of forming v_μ and v_τ is

$$P(v_e \rightarrow v_\mu) = \left|\langle v_\mu|v_e(s)\rangle\right|^2 \tag{3.1}$$

and

$$P(v_e \rightarrow v_\tau) = \left|\langle v_\tau|v_e(s)\rangle\right|^2 \tag{3.2}$$

respectively. The matrix element for $\beta = \mu \; or \; \tau$ is

$$\langle v_\beta|v_e(s)\rangle = \left[u_{\beta1}^* v_1^* + u_{\beta2}^* v_2^* + u_{\beta3}^* v_3^*\right] \times \left[u_{e1}v_1(s) + u_{e2}v_2(s) + u_{e3}v_3(s)\right] \tag{3.3}$$

or, in expanded form,

$$\langle v_\beta|v_e(s)\rangle = u_{\beta1}^*\left[u_{e1}v_1^*v_1(s) + u_{e2}v_1^*v_2(s) + u_{e3}v_1^*v_3(s)\right]$$
$$+ u_{\beta2}^*\left[u_{e1}v_2^*v_1(s) + u_{e2}v_2^*v_2(s) + u_{e3}v_2^*v_3(s)\right] \tag{3.4}$$
$$+ u_{\beta3}^*\left[u_{e1}v_3^*v_1(s) + u_{e2}v_3^*v_2(s) + u_{e3}v_3^*v_3(s)\right]$$

Equation (3.4) is simplified by applying the orthonormality condition $\langle v_i|v_j\rangle = \delta_{ij}$ to obtain

$$\langle v_\beta|v_e(s)\rangle = u_{\beta1}^* u_{e1} v_1^* v_1(s)$$
$$+ u_{\beta2}^* u_{e2} v_2^* v_2(s) \tag{3.5}$$
$$+ u_{\beta3}^* u_{e3} v_3^* v_3(s)$$

The temporal dependence is obtained by expressing Eq. (3.5) in terms of the mass state at $s = 0$:

$$\langle \nu_\beta | \nu_e(s) \rangle = u^*_{\beta 1} u_{e1} \exp\left(-i\frac{T_1 s}{\hbar}\right)$$

$$+ u^*_{\beta 2} u_{e2} \exp\left(-i\frac{T_2 s}{\hbar}\right) \qquad (3.6)$$

$$+ u^*_{\beta 3} u_{e3} \exp\left(-i\frac{T_3 s}{\hbar}\right)$$

4. Application to the $\nu_e \to \nu_\mu$ Transition

We apply the 3 flavor state-formalism to the transition $\nu_e \to \nu_\mu$ in vacuum between two flavor states. In this application we assume the transition $\nu_e \to \nu_\tau$ is negligible and that only 2 flavor states are involved. A 3-state unitary matrix that effectively simplifies the problem so that we only need to consider flavor states 1 and 2 is

$$U(3) = \begin{bmatrix} U_{12}(2) & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.1)$$

where θ_{12} refers to the mixing angle between mass states 1 and 2 in vacuum.

The transition probability amplitude is

$$\langle \nu_\mu | \nu_e(s) \rangle = u^*_{\mu 1} u_{e1} \exp\left(-i\frac{T_1 s}{\hbar}\right)$$

$$+ u^*_{\mu 2} u_{e2} \exp\left(-i\frac{T_2 s}{\hbar}\right) \qquad (4.2)$$

where $\{T_j, j = 1,2\}$ are the eigenvalues of the temporal evolution operator, and

$$u^*_{\mu 1} = -\sin\theta_{12}$$
$$u_{e1} = \cos\theta_{12}$$
$$u^*_{\mu 2} = \cos\theta_{12} \qquad (4.3)$$
$$u_{e2} = \sin\theta_{12}$$

In the conventional theory, T_j is the energy E_j of state j, while in PRD it is the eigenvalue K_j of the mass operator for mass state j. Substituting Eq. (4.3) into Eq. (4.2) gives

$$\langle v_\mu | v_e(s) \rangle = -\sin\theta_{12} \cos\theta_{12} \exp\left(-i\frac{T_1 s}{\hbar}\right)$$

$$+ \cos\theta_{12} \sin\theta_{12} \exp\left(-i\frac{T_2 s}{\hbar}\right) \tag{4.4}$$

$$= \sin\theta_{12} \cos\theta_{12} \left[\exp\left(-i\frac{T_2 s}{\hbar}\right) - \exp\left(-i\frac{T_1 s}{\hbar}\right) \right]$$

and

$$\langle v_\tau | v_e(s) \rangle = 0 \tag{4.5}$$

The transition probability is

$$P(v_e \rightarrow v_\mu) = \left| \langle v_\mu | v_e(s) \rangle \right|^2 \tag{4.6}$$

5. Application to Neutrino Oscillation Experiments

The evolution equation in PRD for a state may be written in terms of the evolution parameter s as

$$i\hbar \frac{\partial}{\partial s} |v_j\rangle = K_j |v_j\rangle \tag{5.1}$$

where K_j is the eigenvalue of the mass operator for mass state j. The evolution parameter dependent solution of Eq. (5.1) in the mass basis for two mass states is

$$\begin{bmatrix} |v_1(s)\rangle \\ |v_2(s)\rangle \end{bmatrix} = \begin{bmatrix} e^{-iK_1 s/\hbar} & 0 \\ 0 & e^{-iK_2 s/\hbar} \end{bmatrix} \begin{bmatrix} |v_1(0)\rangle \\ |v_2(0)\rangle \end{bmatrix} \tag{5.2}$$

where

$$K_j = \hbar^2 k_j^\mu k_{j\mu} / 2m_j = \hbar^2 \left[(\omega_j/c)^2 - k_j \cdot k_j \right] / 2m_j \tag{5.3}$$

In PRD, the components of the energy-momentum four-vector k_j^μ are observables and the mass m_j is a function of statistical values of k_j^μ.

In the flavor oscillation process $v_e \rightarrow v_\mu$, we begin with a pure beam of electron neutrino v_e particles and calculate the probability for formation of muon neutrino v_μ particles. The PRD result for the probability of forming the final state v_μ from initial state v_e is

$$P_{PRD}(v_e \rightarrow v_\mu) = \sin^2 2\theta \sin^2 \left\{ \frac{(m_2 - m_1)c^2}{4\hbar} s \right\} \tag{5.4}$$

$$\equiv \sin^2 2\theta \sin^2 \alpha_{PRD}$$

where s is temporal duration measured by an evolution parameter clock [4]. Dynamical factors are collected in the term α_{PRD}.

Flavor oscillations may be described by quantifying the behavior of two particles. One particle propagates without interaction or oscillation from the source to the detector and serves as a "clock" for the scalar evolution parameter s. The other particle is the oscillating particle. In this application, the source and detector are separated by a distance L.

The most probable trajectory of the non-interacting s-clock particle is

$$s^2 = (\delta t)^2 - \frac{(\delta x)^2}{c^2} = (\delta t)^2 \left[1 - \beta^2\right], \beta = \frac{v}{c}, v = \frac{\delta x}{\delta t} \tag{5.5}$$

The distance δx traveled by the s-clock particle in the interval δt is L, so we obtain

$$s = \frac{L}{c} \frac{\left[1 - \beta^2\right]^{1/2}}{\beta}, \delta x = L \tag{5.6}$$

Substituting Eq. (5.6) into Eq. (5.4) gives

$$P_{PRD}(v_e \rightarrow v_\mu) = \sin^2 2\theta \sin^2 \alpha_{PRD},$$

$$\alpha_{PRD} = \frac{(m_2 - m_1)c^2}{4\hbar} \frac{L}{c} \frac{\left[1 - \beta^2\right]^{1/2}}{\beta} \tag{5.7}$$

The result for the conventional theory denoted by subscript Std is

$$P_{Std}(v_e \rightarrow v_\mu) = \sin^2 2\theta \sin^2 \alpha_{Std}$$

$$\alpha_{Std} = \frac{(m_2^2 - m_1^2)c^4}{4\hbar E_v} \frac{L}{c} \tag{5.8}$$

where E_v is the energy of the ultrarelativistic incident neutrino

$$E_v = \frac{m_v c^2}{\left[1 - \beta^2\right]^{1/2}} \tag{5.9}$$

We combine Eqs. (5.7) and (5.9) and rearrange to simplify comparison with Eq. (5.8):

$$P_{PRD}(v_e \rightarrow v_\mu) = \sin^2 2\theta \sin^2 \alpha_{PRD},$$

$$\alpha_{PRD} = \frac{(m_2 - m_1)c^2}{4\hbar} \frac{L}{c} \frac{m_v c^2}{E_v \beta} = \frac{m_v(m_2 - m_1)c^4}{4\hbar E_v} \frac{L}{c} \frac{1}{\beta} \tag{5.10}$$

The ratio of the dynamical factors $\alpha_{PRD}, \alpha_{Std}$ is

$$\frac{\alpha_{Std}}{\alpha_{PRD}} = \frac{m_2^2 - m_1^2}{m_v(m_2 - m_1)} \beta = \frac{m_1 + m_2}{m_v} \beta \tag{5.11}$$

and the ratio of probabilities in Eqs. (5.8) and (5.10) is

$$\frac{P_{Std}}{P_{PRD}} = \frac{\sin^2 \alpha_{Std}}{\sin^2 \alpha_{PRD}} \tag{5.12}$$

Comparing P_{PRD}, P_{Std} and the dynamical factors $\alpha_{PRD}, \alpha_{Std}$ shows that the PRD model and the conventional theory have the same dependence on the flavor mixing angle θ, but their dependence on dynamical factors differs significantly. If the mass difference between neutrino mass and flavor states is very small and the neutrinos are ultrarelativistic, then $(m_1 + m_2)/m_v \approx 2$ and $\beta \approx 1$. The ratio of dynamical factors $\alpha_{Std}/\alpha_{PRD} \approx 2$ in this case.

The survival probability of the electron neutrino is

$$P_{PRD}(v_e \rightarrow v_e) = 1 - P_{PRD}(v_e \rightarrow v_\mu) = 1 - \sin^2 2\theta \sin^2 \alpha_{PRD} \tag{5.13}$$

in the PRD model, and

$$P_{Std}\left(v_e \to v_e\right) = 1 - P_{Std}\left(v_e \to v_\mu\right) = 1 - \sin^2 2\theta \sin^2 \alpha_{Std} \qquad (5.14)$$

in the conventional Std model. These probabilities are compared in Figure 1. The angle α_{Std} is calculated using $L = 180\,km$, $\Delta m^2 = 7.0 \times 10^{-5}\,eV^2$ and $\sin^2 2\theta = 0.84$ as a function of neutrino energy that varies from 0.1 MeV to 15 MeV [1, Fig. 14.1, pg. 63]. The angle α_{PRD} is calculated using $\alpha_{Std}/\alpha_{PRD} \approx 2$ which is based on the assumption that the neutrinos are ultrarelativistic in vacuum and $\left(m_1 + m_2\right)/m_v \approx 2$.
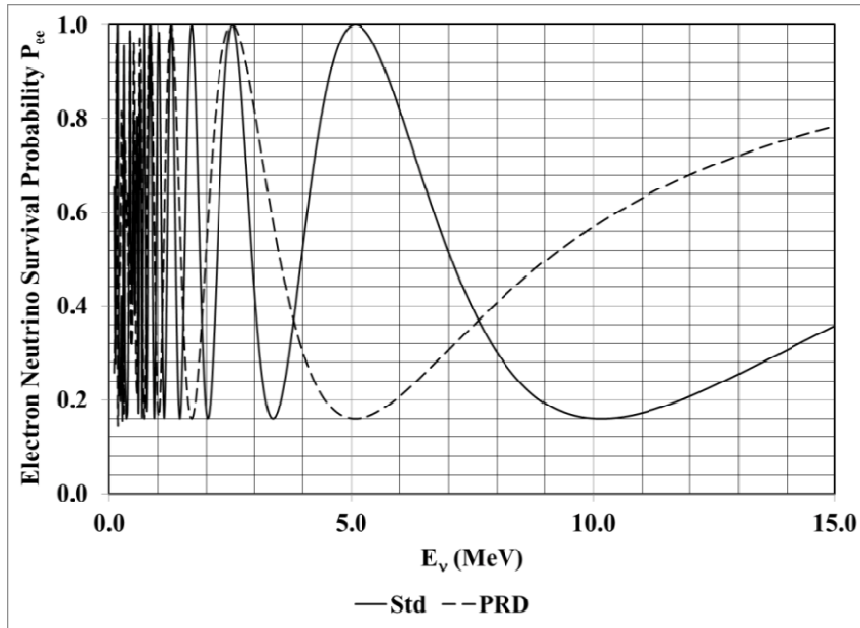

Figure 1. Comparison of Conventional (Std) and PRD Survival Probabilities as a Function of Neutrino Energy

Figure 2 shows survival probability of the electron neutrino as a function of L at a neutrino energy of 10 MeV. The angle α_{Std} is calculated using $\Delta m^2 = 7.0 \times 10^{-5}\,eV^2$ and $\sin^2 2\theta = 0.84$. It is clear from the figures that there are significant differences between conventional (Std) and PRD theoretical results. Figures 1 and 2 are especially useful displays of experimental resultsbecause they provide theory-independent information that can be used to determine the probability of disappearance $P\left(v_e \to v_\mu\right)$ or survival $P\left(v_e \to v_e\right)$ of electron neutrinos.
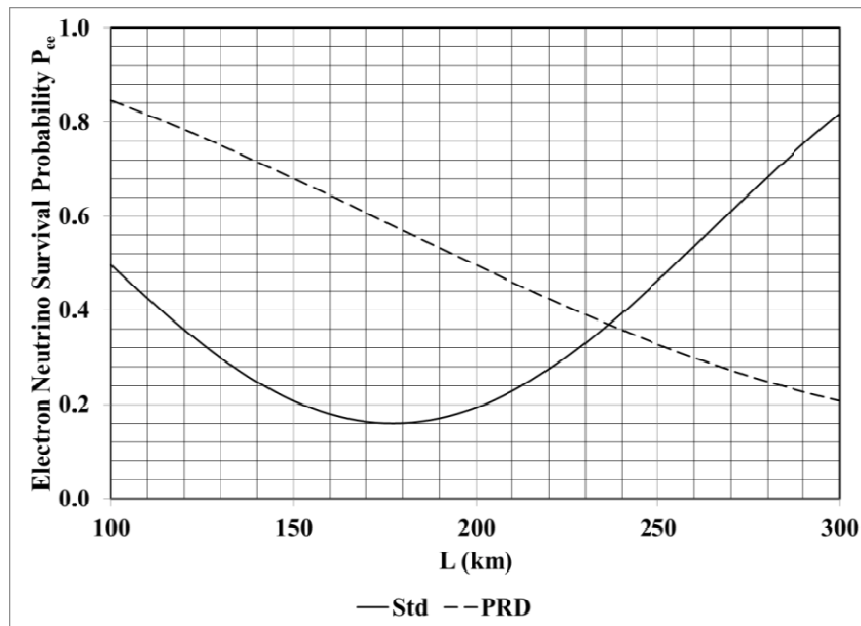
Figure 2. Comparison of Conventional (Std) and PRD Survival Probabilities as a Function of the Distance L between Source and Detector

6. Conclusions

A framework for studying mass state transitions between neutrino flavor states with up to three flavors was presented in the context of parametrized relativistic dynamics (PRD) and applied to the survival of electron neutrinos. The analysis shows that significant differences exist between theoretical results of the conventional model and the PRD model. Current estimates of neutrino masses depend on the mass dependence $\Delta m^2 \equiv m_2^2 - m_1^2$ of the conventional theory. Further work will be needed to examine experimental results within the context of PRD. The result may be a set of neutrino masses that is consistent with the experimental results but differs from the conventional analysis.

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