

The Transition of At-Risk Students to Independent Learners

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Students at risk of failure in high school endure a list of academic hardships and struggles. Failure in school can lead to an attitude of helplessness and defeat. Specific repeated failure in mathematics can leave at-risk students feeling hopelessly behind, and some resign themselves to the idea that they will never succeed. Some students have learned to cope by giving little effort, hiding in the back of a class, and failing to produce work. These behaviors are characteristics of learned helplessness – an attitude of expected failure regardless of effort (Dweck, 1975; Diener & Dweck, 1978; Seifert, 2004). At-risk learners are predominantly placed in mathematics classrooms, which utilize a traditional model of instruction, although these settings may not be the ideal environment for addressing current mathematics standards (Staples, 2007). At-risk students may be hostile to mathematics class in general, and may resist changes from a traditional to a reform-based classroom environment that meets the National Council of Teachers of Mathematics expectations (NCTM, 2000; Yates, 2009; Wachira et. al., 2013). Scaffolding by the teacher is required to gradually transition these students to this reform-based classroom environment that emphasizes concept-based learning and classroom dialogue (Carnine, 1997; Heibert et. al., 1997). The role of the teacher in this process may be the deciding factor to change the attitudes and performance of at-risk students (Wachira et.al., 2013). If the teacher gradually changes the type of support provided during instruction, students may gain confidence, begin to accept more challenges, and further their conceptual understanding. Ultimately, adjustments in the role of the teacher may assist at-risk students in becoming independent learners.

New Horizon High School (NHHS) is an alternative school created to assist at-risk students with the goal of earning a high school diploma. Approximately one-third of the students

of NHHS are failing a mathematics course at the time of enrollment and over half have failed one or more full year mathematics courses. In order to succeed, these students must overcome years of past failure and an attitude of helplessness. In addition to changing their ideas and feelings about mathematics in general, they must learn how to be successful in a classroom environment and further their own learning. The present study examines if at-risk students can successfully learn in a reform-based classroom, and, if so, how this may be accomplished

Literature Review

When students have been faced with repeated failure, some experience the phenomenon of learned helplessness. Students who display learned helplessness choose not to even attempt the actions required to succeed although they may be completely capable. Their perception may be that they cannot overcome their failure – it is beyond their control. When exhibiting learned helplessness, students may be unwilling to engage in classroom activities or dialogue due to the belief that failure is inescapable (Dweck, 1975; Diener & Dweck, 1978; Seifert, 2004). Dweck and Reppucci (1973) found that the performance of some children deteriorated when faced with continued noncontingent failure, although the children possessed the ability to succeed. These findings supported their hypothesis that some students believe that whether they try or not, the consequences will be the same.

In addition to the consequence of failing grades on transcripts, students have experienced failure in everyday classroom activities. Success and failure are strikingly obvious in the mathematics classroom as the answers are observed to be right or wrong by students, and this can lead to a low level of student motivation (Yates, 2009). Students often incorrectly believe that problems should be solved within a few short minutes, and they are simply not good at mathematics if they are not able to quickly find a solution. These negative attitudes of

helplessness can lead to opting out of the work, assuming they will most likely fail regardless of effort. Yates (2009) identified the need for a valid measure of learned helplessness to be used by teachers to identify the characteristics before the behavior becomes entrenched. Teachers were able to reliably identify the behaviors using the Student Behavior Scale designed for elementary and middle school students. This scale measured behaviors such as giving up after correction, becoming discouraged after failing part of a task, and needing help to get started or keep going (Yates, 2009). Students who display these behaviors may be convinced that they cannot control their circumstances and blame their failures on external attributes (Diener & Dweck, 1978). In order for success to be a goal, attitudes must be gradually shifted from concerns about the cause of failure to focusing on the remedy (Diener & Dweck, 1978; Seifert, 2004). This stance poses unique challenges when addressing helpless attitudes of at-risk learners while expecting them to meet on-level expectations and standards.

At-risk learners are historically placed in remedial classes where traditional models of instruction continue to dominate the educational landscape (Staples, 2007). Traditional teaching methodology continues to be promoted for students with differing learning needs (Hudson, Miller, & Butler, 2006); however, this traditional instructional approach may not meet the diverse needs of at-risk learners who have experienced failure with this method. The traditional style follows the same basic pattern: introduction of the topic, explicit instruction and demonstration with examples, guided practice, and assigned homework. The traditional mathematics classroom often involves learning through rote memorization and the performance of procedures, without requiring students to use their prior knowledge to solve problems and make connections. In contrast, according to the National Council of Teachers of Mathematics (NCTM) (2000), developing conceptual understanding is an essential component of learning in a

mathematics classroom. When students gain knowledge through understanding, they are able to retain information and apply this information to new problems and settings (NCTM, 2000). The spiraling failure of at-risk students in mathematics could be due to a lack of both conceptual understanding and sufficient prior knowledge. In order to build their knowledge, understanding, and confidence about their problem-solving skills, these students should be provided with instruction that adapts to their needs, engages them in building understanding, and challenges and supports their learning (NCTM, 2000). Hudson et al. (2006) described the classroom which promotes this type of instruction as the following:

Learning is viewed as an active, social, and interactive process. Students are given opportunities to work together and discuss their mathematical thinking with peers. This enhances their ability to communicate mathematically. In reform-based mathematics, students ultimately are responsible for their own learning. (p. 22)

A reform-based classroom, which aligns with NCTM expectations, has a very different structure from the traditional classroom.

Varying classroom strategies reflecting the characteristics of reform-based instruction have been recommended to meet the needs of students who struggle in mathematics. Smith and Geller (2004) suggest teachers follow a structured procedure for planning and teaching lessons geared toward students with disabilities and those who are at risk for school failure. Although they advocate teaching concepts over procedures, their method follows specific steps beginning with a graphic organizer, followed by examples, and independent practice. This model closely resembles traditional instruction, which may be efficient for students with disabilities, but may not meet the needs of at-risk learners. A second model designed to meet the needs of varying levels in the classroom, the Mathematics Dynamic Assessment model (Allsopp et al., 2008),

advocates informal assessment that addresses three types of understanding: concrete, representational, and abstract. These types of assessment align with reform-based teaching as the student is required to not only correctly complete procedures, but apply their knowledge to new representations and recall prior knowledge to address new situations. While this technique involves authentic assessment, error pattern analysis, and student interviews, the method does not address the actual instruction phase that would occur prior to assessment.

In a third approach, Ferguson (2009) argues that elementary students who struggle in mathematics will benefit from participating in the same problem-solving tasks as the entire class, with adjustments by the teacher to account for different skill levels. This technique involves all students feeling included in an identical task without a division of the classroom according to ability, but it is not specifically designed for secondary students. At-risk learners at the secondary level would benefit from participating in the same tasks as their classmates, but problem-solving skills must be built and developed in order for their performance and self-efficacy to improve. Many at-risk students are considerably behind their peers in grade level and content knowledge, and possess significant gaps and holes in their understanding. To compound this problem, many at-risk students are apathetic or hostile to mathematics class in general (Yates, 2009). These factors present a unique challenge for teachers to create a reform-based classroom while attending to the specific needs of the at-risk student.

With at-risk students, teachers face the daunting task of simultaneous remediation and the teaching of on-level concepts. This dual process should be performed as efficiently as possible, but time constraints should be realistic. Students should not be overwhelmed by an overload of information and content. Teaching them what they need to know without further contributing to their feelings of learned helplessness (Carnine, 1997) requires teacher support. Scaffolding by

the teacher is required to slowly introduce new concepts and procedures while addressing any holes in student prior knowledge.

The teacher of at-risk learners must also scaffold the transition from traditional to reform-based instruction. Students who are accustomed to traditional instruction may be resistant to reform-based instruction (Wachira et al., 2013). Further, as previously stated, at-risk learners often experience traditional instruction (Staples, 2007). The desired reform-based classroom in which learning for conceptual understanding takes place requires the teacher to adjust from simply fulfilling traditional responsibilities of providing information and demonstration to “creating a classroom in which all students can reflect on mathematics and communicate their thoughts and actions” (Hiebert et al., 1997, p. 29). According to Hiebert et al. (1997), teachers should be cautious to provide sufficient information and support, but not encourage complete dependence on the teacher. While assisting students, teachers should be careful not to immediately demonstrate a new technique that differs from what the student is attempting. If assistance is provided too quickly, students can shift from developing their own ideas to trying to satisfy the teacher, which can inhibit their concept and skill development. This balance of being an active participant in classroom discourse while simultaneously providing unimposing guidance can be challenging (Wachira, Pourdavood, & Skitzki, 2013).

The role of the teacher in a reform-based classroom is too often underdeveloped or misunderstood, which may be a contributing factor to the prevalence of traditional instruction (Staples, 2007). An important aspect of the teacher’s role is to create a climate in the classroom where students feel comfortable reflecting on their ideas and communicating with others (Hiebert et al., 1997). Wachira et al. (2013) found that when the teacher’s role was adjusted to a facilitator of mathematical discussion, some students felt more confident about their

mathematical skills. Teachers face a difficult balance of providing necessary information, fostering emerging problem-solving skills, and creating an ideal learning environment. This balance is critical for at-risk students attempting to become autonomous learners.

The purpose of the present study is to examine how at-risk students can successfully learn in a reform-based classroom, and, if so, how this may be accomplished. Adjusting the role of the teacher by gradually altering the style of the delivery of the lesson may be the critical factor. By focusing on the ratio and type of student-to-teacher involvement (Paniatti, 2009), students may gradually depend less on the teacher and more on themselves while problem solving. This method of merging explicit instruction in a traditional setting with student-initiated independent learning (Hudson et al., 2006) has the potential to assist at-risk students in becoming autonomous learners. Efforts to gradually make the transition from traditional instruction to a reform-based classroom will be analyzed to address the following questions: 1) How can at-risk students best be assisted in the transition to a reform-based mathematics classroom? 2) How will the students' attitude, perception of their abilities, and performance change throughout the transition?

Methods

Research Design

Action research, a type of research in which teachers systematically examine their instructional practices (Hendricks, 2013), provided the model for this study. Teachers participating in action research follow a process which Hendricks (2013) describes as systematic inquiry based on ongoing reflection. The action research process creates a cycle which does not end, but spirals from step to step. This cycle of systematically and intentionally setting goals, creating action plans, observing and thinking about results, and then applying this reflection to modify existing and/or develop new action plans (Dinkelman, 1997), allowed for a fluid and

ongoing reflective cycle for this study. Both qualitative and quantitative forms of data were collected and analyzed with the goal of informing future action plans in the action research cycle.

Setting

Joshua Independent School District (JISD) encompasses 75 square miles of rural and suburban area inside of Johnson County. Joshua is a single high school town, with a 2013 graduating class of 250 students. During the early 1990's, JISD realized homeless and transient students were having a difficult time meeting attendance and academic obligations. To meet the need of making education accessible for these students, the district opened a campus called the Joshua Accelerated Learning Center in portable buildings across the highway from Joshua High School. Students applied and were accepted based on individual circumstances. The school became a safe place for students to learn, catch up and graduate with their class, recover from trauma or addiction, and feel like they were part of the high school experience for the first time. As the school grew, the district constructed a permanent location as a self-contained one-wing addition to the Joshua High School Ninth Grade Campus. Today, the newly named New Horizon High School (NHHS) serves approximately 60 to 65 students and boasted a 2013 graduating class of 57.

The staff at NHHS consists of five full-time teachers, one in each of the content areas of mathematics, science, social studies, English, and career and technology education. Students range in age from 15 to 19 years old, and the school serves students enrolled in grades 9 through 12. The process for entrance begins with a student-initiated application. To qualify for enrollment, the student must be classified as *at-risk* by meeting one of several categories: homelessness, pregnancy/being a parent, on juvenile probation/parole, recent release from an inpatient treatment facility, two or more years behind academically, multiple disciplinary

placements, or multiple and repeated failed state assessments. Students are required to write an essay defining the issues they feel are contributing to their difficulties in high school and explaining why they would be an asset to NHHS. After careful consideration of the application and essay, the parent and child are interviewed and a decision regarding acceptance is made.

Because the students themselves are requesting the transfer to NHHS, most are appreciative of acceptance and value the opportunity they have been given. The school is completely self-contained in one hallway including all five teachers' classrooms, and there is a family atmosphere. Everyone at school has a reason for being there, and bullying and common discipline problems are virtually non-existent. However, because of each child's experiences throughout their life that have brought them to be at-risk, the task of mastering difficult content and catching up academically can be exceptionally difficult. Students are not accustomed to participating in the traditional give and take of an active and productive classroom. In fact, even the most grateful students can rebel and shut down emotionally when presented with academic challenges.

In the mathematics classroom at NHHS, the students have extremely varying abilities within a single class, vast holes and gaps in prior learning, and often possess a mindset of fear and failure. Most students have been unsuccessful in mathematics throughout their entire educational experience, and the daily trials of teaching difficult content to students who are not interested in school is extremely challenging. Most of the students in the classes have failed one or multiple grade levels, mathematics courses, and/or standardized tests. For example, 69% of students enrolled in the 2012-2013 school year transferred with failing mathematics grades, and 53% of students enrolled failed the most recent mathematics course they attempted at their previous high school. Fifty-five mathematics students have an accumulated total of 39 failed

mathematics courses posted to their transcripts during their high school careers. Of these 39 mathematics course failures, 30 were attempts at Geometry. Many of these students have only experienced a teacher-led traditional classroom involving learning through rote memorization with a goal of attaining procedural knowledge. This repeated academic failure has conditioned some students to believe they cannot achieve in mathematics. Due to the aforementioned challenges that the students have with geometry, this study examined an attempt to shift to a reform-based learning environment in one geometry class.

Participants

The participants of this study were the mathematics teacher, who was also the researcher, and the 16 students enrolled in Geometry at New Horizon High School. Parental permission and student assent were obtained prior to data collection, and all documents were returned to a colleague. The students participated in learning activities that were completed in groups. These activities, referred to as labs, were during normal classroom time on Tuesday and/or Thursday. Monday, Wednesday, and Friday were devoted to fulfilling the district curriculum requirements of lessons and assignments. During these lab times, the student and teacher dialogue was audio recorded and student artifacts were collected. The social studies teacher who collected the parent permission and assent documents grouped the students and chose two focus groups. The dialogue of all whole-class discussions and of the focus group was transcribed; however, the interactions of all four groups in the classroom were audio recorded. The central effort of the intervention was focused on the ratio of student-to-teacher involvement and changes in the type of teacher involvement over time. The transition from a traditional to reform-based classroom was implemented over the period of one semester by gradually adjusting the role of the teacher during labs.

Obtaining consent proved to be a challenge as only eight out of sixteen students returned signed documents, but my colleague was able to create two groups of four to participate in the study. I decided to focus on both groups – eight students in all. Because the population at our school fluctuates, this would allow me to continue collecting data if one or more students were to withdraw. Also, by focusing on eight students I was able to re-group them or partner them as necessary. Seating and group make up were a challenge and change was a good classroom management tool. Pseudonyms have been used throughout. The eight students comprising the focus group included:

- Travis – a 17 year-old senior repeating Geometry – his third attempt. Travis was overconfident about his abilities but was willing to help others. He was not afraid to answer questions and take the lead.
- Mike – a 16-year old first-time sophomore. Mike gained both eighth grade and freshman credits (including Algebra I) through a point-and-click computer program in a self-contained lab on the middle school campus. Mike was not very attentive in class and attending NHHS was not his choice but that of his parents. He had not been placed in a traditional classroom setting due to two years of disciplinary and credit issues.
- Becca – a 17 year-old sophomore attempting Geometry for the first time. Becca was retained during middle school and elementary school. She transferred to NHHS due to her anticipated graduation date after her 20th birthday. She struggled with basic mathematics skills and had failed to pass the Algebra I standardized test after several attempts.
- Jennifer – a 16 year-old junior repeating Geometry. Jennifer did not gain credit in any of her sophomore core classes during her 10th grade year. Her plan included graduation

within the year and she worked very hard. Jennifer was unsuccessful on standardized tests in all subjects; however, she was diligent in her studies and participated in the classroom.

- Lexie – a 17 year-old junior repeating Geometry. Lexie chose to attend NHHS after failing to accrue credit in a traditional high school setting. She was apathetic in class and not particularly interested in the content.
- Cassie – an 18 year-old junior and special education student repeating Geometry. Cassie was not confident in her abilities and had received content mastery services throughout her educational career.
- Jeff – a 15 year-old sophomore and special education student attempting Geometry for the first time. Due to ongoing discipline issues throughout elementary, middle school, and as a freshman, Jeff was placed at NHHS with the hope of receiving a high school diploma. He was very confident in the classroom and his mathematics skills were strong.
- Nicole – an 18 year-old junior repeating Geometry for the second time. Nicole had previously failed several drug tests and missed extended periods of school for drug-related issues. Her basic mathematics skills were not strong, but she was familiar with the material in the course.

During the course of the study, four students were withdrawn at different times under various circumstances. At the conclusion only four students remained: Nicole, Jennifer, Becca, and Travis.

Phase One

To begin, I maintained a 50:50 ratio of independent work to instruction and support (Paniati, 2009). For example, warm up problems, which related to the day's activities, for the

students to be worked together were provided when the students entered the classroom. I then provided the students with their lab assignment by explaining the instructions and working through examples. In groups, the students then attempted to work on a section of the lab, after which the groups came together to evaluate their results. After the class examined their solutions, the groups were assigned the next problem or section of the lab to solve together. After working for a short time, the class came together again and evaluated all results. This back-and-forth approach was used to keep the students on track and provide feedback for their efforts at regular intervals. I could evaluate their potential misunderstandings and conceptual deficiencies. During this type of instruction, I was looking for student interaction within the groups and the type and frequency of student questions. Were the groups able to begin a task independently? Were they successful at this attempt? Whether students were willing to attempt to begin a task independently, and whether they were successful at this attempt, were two very different evaluations. If students were willing to attempt a task independently they were displaying confidence in their abilities and could possess understanding. Regardless of whether they were successful in this attempt, the willingness to try was indicative of progress. When I began to see growth in both areas - willingness to attempt a problem and some success in the attempt - the transition to the next phase was planned. At the completion of phase one, I provided an anonymous online survey for students regarding their perception of their abilities and growth in Geometry. The responses to this survey were considered while planning the pace of the next phase. Figure 1 illustrates the action research cycle that was repeated with each phase of implementation. During each phase I analyzed classroom dialogue and student performance to determine the pacing and timing of the change in the classroom roles. A student survey served

the dual purpose of completing and initiating each cycle as well as informing me of the students' perceptions.

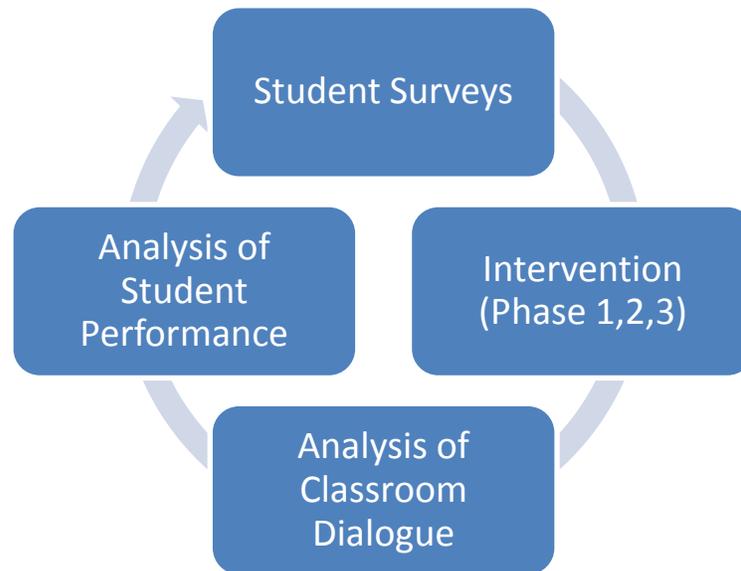


Figure 1. The Action Research Cycle

Phase Two

The second phase of the intervention was a gradual transition into a new format for labs. I maintained the 50:50 division of time; however, the students evaluated a problem or idea and discussed their strategies *before* a new concept was introduced (Paniati, 2009). A lab involving this instructional format began with a task or problem. The task related and built upon what the class has been discussing in previous lessons. I assigned the task and allowed the students time to discuss strategies with their group. My role was to monitor and support student discussions and evaluate the type and frequency of their questions. Their questions were evaluated to determine: Did they continue to see the teacher as the only source of information or were they using each other as resources? Were they beginning to make an attempt – even if it was not

correct – independently? Initially, I provided direction to steer some groups, but the goal was for the students to begin to develop strategies and make attempts without specific guidance. After students had been allowed time to work together and develop a product to present, the class came together and evaluated each other's work. The remaining class time (approximately 50% of the period) was used to teach the concept which built on the students' correct and incorrect attempts. As the students' discussions within their group increased and they were more confidently and accurately working on a task, the next transition was planned. Near the completion of this phase, an anonymous survey was administered. The responses to this survey helped to inform the timing and pace of the research cycle.

Phase Three

The final phase of intervention involved a shift in the ratio of student-to-teacher involvement to a 75:25 split (Paniati, 2009). The students were given a task and provided with the majority of class time to work together. I provided limited set up for the task or problem and took on a facilitating or advisory role. Support was provided when necessary, but students were allowed to discuss the task and begin without detailed explicit instruction. Their discussions were monitored to determine how willing and confident they were to attempt the activity, their accuracy in applying concepts to new problem situations, and their ultimate success in their problem-solving attempts. I was available to assist, but tried to ask supporting questions, and encourage them to evaluate their work further instead of providing an answer or even steering the students in a particular direction. The students were responsible for communicating their ideas to the class and eventually evaluating the ideas of other groups. The remaining 25% of the class time was used to solidify the concepts underlying the task. Ultimately, the students displayed increased and refined communication skills. Questions to be considered during this

phase included: Were they capable of staying on task for 75% of the class period? Could they come to a conclusion – successful or not? Was the number of successful attempts increasing? How well were they applying their conceptual knowledge to new tasks? A final survey was taken to evaluate student attitudes and the perception of their performance throughout the semester.

During all phases of the research cycle, teacher created formative and summative assessments were given. A district benchmark was administered in October. Student performance and written work on these assessments was evaluated to determine whether they were capable of applying their knowledge. Their scores on assessments as well as their answers to the anonymous surveys were a factor in the pacing of the interventions. Additionally, field notes and transcription of classroom dialogue informed the action research cycle.

Data Sources and Analysis

Transcriptions of classroom dialogue, field notes, and student artifacts from labs, assessments, and district benchmarks served as data sources. Each lab was audio taped and the focus groups' dialogue was transcribed. I recorded detailed field notes during the conference period immediately following Geometry class documenting activity during each lab including the students' frequency and type of questions asked as well as teacher perceptions and observations of student performance. Anonymous student surveys were given at the beginning of the semester to gauge student attitudes, perceptions, and prior experiences in mathematics (Appendix A). Different surveys addressing potential changes in student attitudes and perceptions were administered during each intervention cycle (Appendices B and C). Each survey was administered anonymously through a survey website the students accessed during their technology class period with help from the career and technology teacher. The answers to

these surveys, in conjunction with the other data sources, helped to determine when to begin each new phase of intervention.

The data gathered from transcriptions of audio recordings, field notes, and student surveys were coded using the constant comparative method (Glaser & Strauss, 1967). This method requires data to be examined and compared for emerging codes. The data were analyzed as they were collected. Codes shown in Table 1 were generated based on the analysis of the data. As more data was analyzed, new codes were compared to existing codes, with codes merged if appropriate. Since the data were coded as collected, the analysis helped to inform the action research cycle.

All lab assignments and unit tests were collected and closely examined. The analysis of these was based on the amount and quality of work shown and the accuracy of attempts. These artifacts helped to determine if the students were successfully applying the knowledge they had learned to familiar and new problems or situations. Evaluation of their attempts or non-attempts at every problem served as an assessment of their confidence level. The scores on these assessments served as evidence to address the aspect of the research question involving their performance.

Finally, district benchmarks were required, and their evaluation further informed the research cycle. Due to the lack of a standardized test specific to Geometry, teachers were allowed to create their own benchmarks based on material covered in class. Benchmarks were evaluated not only for correct answers, but also for attempts and worked-out problems. If the students were becoming more confident in their abilities to attempt difficult problems, they were on the path to becoming independent, autonomous, life-long learners.

Table 1

Frequency of Codes by Phase

| Code | Phase One (<i>n</i> = 8) | Phase Two (<i>n</i> = 8) | Phase Three (<i>n</i> = 6) |
|--|------------------------------|------------------------------|--------------------------------|
| Self-defeating Statements | 7 | 1 | |
| Same Student Asking Repeat Question | 4 | 2 | |
| Different Group Asking Repeat Question | 8 | 3 | |
| Incorrect Response | 2 | | |
| Insufficient Knowledge of Supporting Standards | 3 | 1 | |
| Student Helping Others Within the Group | | 2 | 5 |
| Insecure Statements | | 3 | 1 |
| Requests Teacher to Review Work | | 3 | 2 |
| Expresses Confidence in Abilities | | | 3 |
| Correct Verbal First Response | 1 | 1 | 5 |
| Requests Confirmation for Correct Work | | 2 | 4 |
| Discussion Within Group Prior to Help Request | | | 6 |
| Discussion with Other Groups Prior to Help Request | | 3 | 3 |

Findings

Phase One

During the first three weeks of instruction, while waiting on consent documents to be returned, the class reviewed Algebra basics such as solving equations and graphing linear functions. The 16 week-long semester included three weeks to obtain documents, one week for benchmark testing, one week for December administration of standardized testing, and one week for final exams. This left the class with nine full weeks of instruction which was divided into six Geometry units (Appendix D). These units included the following topics: triangles, distance and

measurement, area and perimeter, circles, three dimensional prisms, and logic. Each unit contained between three and five lessons, a review, and a test. In order to fit the entire half of Geometry into the fall semester, our lab activities were limited to one day per week. These labs took place either Tuesday or Thursday during the normal Geometry class time.

After focus groups were formed, I began the first phase with activities that aligned with the curriculum. Because most of the students were repeating Geometry, they were familiar with the first unit on Geometry basics. Typically we would begin the class with vocabulary such as point, line, plane, and angles, however; we began with triangles and incorporated initial Geometry vocabulary and basics into the lessons for review as the course progressed. The labs in phase one involved describing the Pythagorean Theorem, using the Pythagorean Theorem to find distance on a coordinate plane, applying basic measurement skills, and using ratios to compare similar figures.

Initially, students expressed self-defeating statements such as, “I give up,” or “I can’t.” These self-defeating statements occurred most often when students were trying something new. For example, students were attempting to write the Pythagorean Theorem in their own words and Mike became frustrated several times.

| | |
|----------|--|
| Jeff | a and b are sides. |
| Jennifer | No, a is a leg |
| Nicole | And b is a leg, c is the hypotenuse. |
| Mike | I give up. |

Although Mike verbally expressed that he gave up, he continued to observe and attempt to be a part of the activity at his table. Mike and Jeff both continued to express self-defeating language as they worked further through the activity with their group.

Jennifer Okay so a squared, and then b squared, and then we need to show that c is the square root.

Nicole Can we just draw it?

Teacher Yes, but you need to be ready to explain it.

Jeff I give up.

Mike I give up.

A second recurring code involved students in the same group asking the same question multiple times, even after I had answered their question.

Mike Is there like an easy way? Can't you just draw a line?

Teacher Well you could, but try to think about driving on a street.

Nicole Do we have to draw a line?

Teacher You do need to draw your path. Think about driving on a street.

Nicole and Mike were attempting to draw a measurable path between two points on a coordinate plane. The task required them to drive along vertical and horizontal lines instead of cutting across the squares. When Mike asked if he could just draw a line, he visually demonstrated a straight path between the two points. Although I clarified for him to think about driving on a street, Nicole immediately asked the same type of question and made the same motions on her paper showing the direct distance between the two points. When this type of questioning occurred within groups, I began to attempt to provide the same answer. I hoped this would encourage them to work together and seek each other as a source of information.

The same type of questioning occurred across groups. Groups would ask similar or even identical questions regarding the same tasks.

Teacher Can you make coordinates for the houses? What if the school is your origin?

Cassie I don't even know what I'm doing here.

Teacher When I say "origin" what does that mean on my graph?

Cassie points to the origin.

Teacher I know, but using numbers – what does the origin mean? So what if you let your school be (0, 0) what does that tell you about this line and this line?

Cassie x and y ? So rise over run?

Teacher Yes, but you're thinking slope. When we just want to name the point, x comes first and then y . Try to find the coordinates of your house and your friend's house.

After Cassie addressed the incorrect procedure of rise over run when plotting points, Nicole and Becca - working in two different pairs - asked almost identical questions. Although I had addressed Cassie's question using the coordinate plane on the board as a reference, the two girls did not seem to hear or comprehend my explanation.

Nicole Okay so I don't get this. Don't I need to go up first and then over?

Teacher When you're doing slope yes, but when you are just finding your points the x comes first.

Becca Is it rise or run first? I can't remember.

Despite my two explanations, there continued to be questions about plotting points on the coordinate plan. This was indicative of a third code I found in the data involving weak supporting-standards skills. In many cases, background knowledge was insufficient to

successfully complete activities without help to even get started. I began to assess what basic background skills were needed prior to each lesson and incorporate those skills into warm-up problems.

During phase one, even if the students had completed warm-up activities relating to the essential skills necessary for the day's activities, they continued to struggle with basic concepts. Before doing activities related to measurement and ratios, several ratios were created and changed into decimal representations. Regardless, changing a fraction into a decimal proved to be challenging.

Teacher Create a ratio of the two measurements – then divide the top by the bottom.

Cassie What if it isn't a decimal? Just divide?

Lexie I'm not sure how to do this and round it.

Teacher You have to divide it out.

Even after receiving the explicit explanation about creating the ratio and dividing the numerator by the denominator, several students continued to have problems and needed more clarification.

Teacher Travis, how tall are you from your feet to your head?

Travis 61 inches

Teacher How long was it from your feet to your belly button?

Travis 39 inches

Teacher So if I told you to give me the ratio as a decimal, what would you do?

Travis Divide?

Another unexpected representation of poor background skills was the act of measuring. Measuring each other's height on a large piece of paper proved to be very difficult, and students often had to go back and correct their measurements.

Jeff I know I'm 5'7'' but it says I'm not.

Nicole I'm taller than you, I don't think that's right.

The majority of the class period – during this lesson intended to develop the idea of similar figures and equivalent ratios – was spent measuring and re-measuring height. The lack of background knowledge and poor supporting mathematical skills created a big challenge. As the teacher, I had to decide how much time should be spent reviewing very basic skills typically taught in elementary and middle school. Ultimately, they needed to stay up with the pace of the course in order to complete it and move on to another mathematics class as most of them were behind schedule to graduate on time.

Although the students struggled through some of the labs, the students performed well on the first test. The Unit One test (Appendix F) included twelve multiple-choice questions involving the use of the Pythagorean Theorem and finding missing sides of triangles with special right triangle ratios. Most of the students that were part of the focus groups were quite successful, as shown in Table 2, and three students answered every question correctly. All students showed some type of work on their tests except for Travis – one possible explanation could be that Travis was skilled with a calculator and was able to complete the problems showing minimal work. Because this was Travis' third attempt at this course, and he had shown that he understood the material, I did not discuss his lack of work shown on his test with him. Alternately, Becca showed extensive work on each problem but seemed to struggle and wrote

question marks in around the problems she did not understand. Mike received the lowest grade of the group at 70%.

After the confidence booster of the Unit One test, the Unit Two test (Appendix G) incorporated open-ended response questions. The topics in Unit Two included distance and midpoint, basic triangle vocabulary, similar figures, and indirect measurement. Because I was evaluating tests to determine not only correct answers, but also willingness to attempt problems, score alone did not assure me of their progress. Becca again showed the most detailed work, but only answered twenty out of twenty six questions correctly. I was able to follow Becca's work, and she continued to make mathematical errors. However, even after several whole-class activities using Pythagorean Theorem to find distance on a coordinate plane, Becca was the only student who used this approach and correctly answered both distance problems. Travis, followed by Jeff, showed the least amount of work; however, they only answered one and two problems incorrectly, respectively. Only one student, Lexie, answered fewer than 70% of the questions correctly. She showed no work on the problems involving distance, and failed to use either the distance formula or Pythagorean Theorem, even though a coordinate plane was provided with designated points. Because most of the students did show work, even if their attempts were unsuccessful, I felt they were ready to begin transitioning to the next phase.

As the self-defeating speech began to decrease and students were continuing to attempt new tasks I administered the first anonymous online survey. The results shown in Table 3 were both encouraging and discouraging, but provided insight into the students' attitudes about the class. I expected more students to express their dislike for math and was surprised they did not. Possibly because some of the students had failed Geometry previously the majority of students expressed that they felt Geometry would be "hard", and they were worried about passing.

Although no one selected the response indicating they “hated” Geometry, 8.33% selected the response indicating they felt they would fail “for sure.”

Table 2

Test Scores as Percentages

| Student | Unit 1 | Unit 2 | Unit 3 | Bench- mark | Unit 4 | Unit 5 | Unit 6 |
|----------|--------|--------|--------|----------------|--------|--------|--------|
| Becca | 75 | 88 | 80 | 55/75 | 86 | 100 | 85 |
| Cassie | 60 | 84 | 77 | 85 | -- | -- | -- |
| Jeff | 97 | 92 | 100 | 95 | DAEP | GED | GED |
| Jennifer | 97 | 94 | 81 | 95/100 | 86 | 93 | 95 |
| Lexie | 67 | 65/79 | 98 | 60/70 | 82 | 84 | 84 |
| Mike | 70 | 77 | 77 | 64/73 | 91 | 72 | 50 |
| Nicole | 90 | 96 | 90 | 85 | 95 | 100 | 90 |
| Travis | 105 | 96 | 96 | 91 | 91 | 100 | 75 |

Note. DAEP refers to a Disciplinary Alternative Education Placement. GED refers to General Education Development Testing. Both initial score and score after corrections are included.

The survey included a write in component which proved to be somewhat more encouraging. When asked about the hardest part about mathematics some responses included:

- Solving the word problems
- All the steps and to remember it
- Before, EVERYTHING

Although one student answered “everything,” I was pleased that some were able to determine a specific skill or issue in which they were weak. Students were asked to describe a typical math class and whether all classes are the same or different.

- Some different cause this school is like one on one, and to me it is so much easier.

- It's different. My teacher would use advanced words and I could not understand even if I asked for help.
- Before I really couldn't and didn't understand it now it's super easy and I don't know why I thought it was difficult before.
- Algebra I and Geometry I felt like the teacher just didn't teach very good. And when I would ask for extra help on the side they wouldn't give it to me.

The responses indicated that the students were able to articulate their issues and difficulties in previous classrooms. The student's response regarding "advanced words" motivated me to reflect on my word choice, and when listening to the audio recordings I analyzed my use of academic vocabulary. Most importantly, their attitudes shown in the survey were encouraging and supported my decision to move forward with the transition to phase two.

Previously I felt that perhaps I had been providing the students with too much explicit process help. However, after reading the survey responses, I realized that providing the help was essential to build their trust and encourage their growth. The students were more willing than I expected to take on any task presented, even if they were not willing to attempt it on their own and asked multiple and repeated questions. In some cases they lacked the background knowledge needed to begin independently, but they were willing to ask for help. These common ideas emerged as codes from the data analysis. As I evaluated data from all sources I developed additional codes throughout each phase.

Table 3

Initial Student Survey Responses

| Answer Choice | Percentage of Student Responses (<i>n</i> = 8) |
|---|--|
| <i>Math is...</i> | |
| My favorite subject. | 16.67% |
| Ok, I don't really have any problems with it. | 66.67% |
| Very Hard. I don't like Math | 16.67% |
| Impossible. I hate it. | 0% |
| <i>I think Geometry will be...</i> | |
| Super easy. | 8.33% |
| Challenging, but I think I can handle it. | 33.33% |
| Hard. I'm worried about passing. | 58.33% |
| Impossible. I will fail for sure. | 8.33% |

Phase Two

A subtle difference in students' language began to occur during phase two. The previous self-defeating speech began to slowly transition into what I decided to code as *insecure statements*. Instead of giving up and expressing that they had no idea how to do something, they began to make questioning statements. For instance, Jeff was indecisive about his technique to find the area of an irregular shape using clear transparency grid paper.

Jeff I'm about to find the area. Mine is an octopus. Can I just count the squares?

Teacher Sure. But keep in mind some might not be full squares.

Jeff I know.

Teacher Then you're on the right track.

After working for a few minutes

Jeff I'm not sure if I'm doing it right.

I had intended to create a structure with the activities in phase two that allowed for the students to attempt problems for 50 percent of the class time followed by whole-group instruction of the concept. However, the above dialogue is an example in which I provided more assistance than I planned. As I reviewed the data, I realized I needed to better maintain my role of monitoring and support. As I attempted to scale back my help, students did begin to seek help from others. Asking another student a question, as well as offering help to other students, became a coding characterization within phase two. For instance, two girls were working together on strategies for finding composite area.

Becca I have something different.

Teacher How did you find the area?

Becca I haven't yet, I'm helping Nicole.

Teacher Okay, so how did ya'll find Nicole's?

Nicole We counted all the squares and the half squares and then added them together.

Becca became more enthusiastic about showing her strategies and discussing her work. Several students began to express their confidence and think through their strategies, even if they were initially insecure.

Travis I'm not sure what to do.

Teacher What Jeff is doing is going to help find the area.

Travis Is he measuring? We can use the plastic. I got this.

Teaching supporting skills continued to dominate most of my time during lab activities; however, students were beginning to use their supporting skills correctly when applied to new situations. For instance, during a lab activity, students were provided with a short scenario and required to measure actual distance. A particular problem involved measuring the distance from the door of the school building to a certain numbered parking space. Students were provided with a meter stick, string, and other various objects they chose to measure distance. All groups correctly measured small distances and then added them together to get the total distance – a skill they had struggled with during phase one as they measured their height.

District benchmarks were administered during this phase. Because there is no longer a Geometry STAAR test, the district allowed teachers to create their own benchmark which assessed topics from Units One and Two. Some of the problems on the benchmark were nearly identical to problems students had answered previously. The benchmark consisted of 11 questions – three multiple choice and eight free response. Lexie and Becca had difficulty with the exam and their attempts required corrections. Although they both began some problems correctly, and seemed to follow the correct procedural steps, they were unable to complete the necessary steps to find the correct answer. Lexie received modifications on her test. She was allowed to use her notes and to ask procedural questions; she was able to answer seven out of 11 questions correctly. Becca answered the first question correctly, and then answered six consecutive questions incorrectly. When asked to find the midpoint given two points, she applied the slope formula. Although her work was correct, the process was not. Even though both Becca and Lexie required tutoring and corrections to bring their grade up to passing, I was encouraged by their attempts. Mike failed to attempt two problems and failed to show work on

two more which he answered incorrectly. Mike did not seem to be increasing his attempts; however, the problems he worked out were answered correctly. . Nicole showed improvement in her willingness to work out problems. She attempted every problem and answered nine out of 11 correctly. She showed work on all problems and followed procedures correctly. She made mathematical errors which resulted in incorrect solutions for two problems. Travis and Jennifer each answered ten out of 11 questions correctly. Jennifer showed extensive work and attempted every problem, while Travis showed less work. Due to this being Travis' third attempt at the course, I did not require him to rework any of the problems on his test. I felt confident he was working through the answers using his calculator and did not need to show each step. Jennifer left very detailed work on her test but set up a proportion incorrectly for a problem involving missing measurements of similar figures resulting in her one incorrect response. I felt that all of the students except for Mike were making an effort to attempt every problem, even if they were not sure how to complete the procedures to find the right answer.

Although the students were occasionally continuing to view me, the teacher, as the primary source of information, they were beginning to work more cohesively within their groups and showed growth in their problem-solving skills. I administered the second online anonymous survey to support my decision to transition to phase three. The survey results in Table 5 were very encouraging. When asked what their thoughts were about mathematics now, all of the students replied, "It's getting better. I feel more confident." A majority acknowledged that Geometry was "A little hard," but they all expressed confidence that they were understanding and "getting it." When asked for an open-ended response about what they felt they were good at in Geometry one answered with specific skills stating, " $a^2 + b^2 = c^2$), fractions." I was pleased that this student was able to name a specific skill, and that this was a

skill on which we had been working. Also encouraging was the response, “anything as long as I pay really close attention and ask for help.” Responses like these encouraged me to move forward with a transition to phase three.

Phase Three

Due to time constraints following a longer than expected consent process and a short semester (16 weeks), phase three was the most brief. I began transitioning to phase three while working on a unit on the volume of three-dimensional figures. One of the most significant changes in written student work was noticed during phase three. Because most of the questioning during phases one and two were answered verbally, students would often turn in lab papers leaving some of the questions unanswered. However, during phase three, Nicole and Becca attempted every written question on a prisms and volume lab. Although their answers were simple and sometimes incomplete, the attempt to complete an entire lab and answer each question was noted as improvement in independent learning. In fact, only one group did not complete every question on that particular lab.

The lab activity involving volume of prisms highlighted the students’ growing communication skills and confidence in their abilities. For instance, Becca read the instructions out loud and then began to fold a piece of paper into a prism. Nicole encouraged Becca and directed her.

Becca We have to make a prism.

Nicole We got this. Fold it half way.

Becca folded the prism according to directions.

Nicole We did it! We have a prism.

Table 4

Second Student Survey Responses

| Answer Choice | Percentage of Student Responses (<i>n</i> = 7) |
|---|--|
| <i>When I think about Math now...</i> | |
| I still love it. | 0% |
| It's getting better. I feel more confident. | 100% |
| It's still hard but I think I will pass. | 0% |
| Impossible. I still hate it. Nothing has changed. | 0% |
| <i>I think Geometry is...</i> | |
| An easy class, no problems. | 33.33% |
| It is a little hard but I'm getting it. | 67.67% |
| It's really hard. I'm struggling. | 0% |
| Impossible. I don't get it at all. | 0% |

The students also took part in a discussion about whether or not the prisms they created would hold the same amount of popcorn. Not only did they discuss their ideas with their partner or group, they talked with students at other tables as well without involving me in their conversation.

- Jennifer Okay, so one is 11.5 inches and one is 8 inches high
- Becca Okay, it's got to stand up.
- Lexie No, it won't hold the same because it's not the same!
- Jennifer Yes, they will hold the same amount.

Mike Yes, she's right they will. Wait, I think this one will hold more.

In addition to their increased reliance on each other and willingness to take risks and make decisions, they began to show understanding of the mathematical concepts. This was very encouraging as I felt that I had spent most of the lab and activity time reviewing eighth grade essential knowledge and skills in order to prepare them to better handle the actual lessons and assignments in the curriculum.

Travis Why don't we just do the equation to find out which one will hold more?

Teacher We're going to do that, but let's estimate first.

Travis But they have the same lateral area, so they will be the same?

Teacher Is lateral area the same as volume? Try your experiment and see what happens.

Although Travis' logic was flawed about the volume of the prisms, he was using prior knowledge of lateral area from a unit we had just completed. Finally, the students began to get excited by their success and seek approval when they knew they had completed a task correctly, as Mike exclaimed: "Mrs. Couey I was right! I was right!" The class average for the Unit Five test (Appendix J) was 92%. Students showed their skill applying formulas and using their formula charts.

As the semester ended, I began to teach a unit introducing logical thinking and proofs. Learning proof has been a challenge for my students in the past so the unit is strategically placed after basic Geometric concepts and prior to the study of more in-depth theorems and properties, which require logical analysis. Because this can be a stumbling block for students, performance on tests declined as expected. The mean grade on the test covering this material was 80 percent - a 12-point drop from the class mean of the previous test. Most students answered multiple-

choice questions correctly when given a conditional statement and asked to identify the converse, inverse, or contrapositive, but students struggled with an actual algebraic proof. Although the steps to each proof were provided in a bank on the test, many students were unable to order the steps correctly and this led to poor test grades.

The link to the final anonymous survey was digitally shared with all students remaining in the district at the end of the semester. The change in attitude about mathematics and their honest view of their own performance was seen in the responses. Although the responses shown in Table 6 were limited, all of the students responded that when they thought about mathematics now, they were doing a lot more than they thought they could. Only one student opted to write in a response for something they needed to work on in mathematics, expressing a need to work toward higher test scores.

Not all students were able to take part in the final survey. Lexie changed foster homes and transferred to a new school, Cassie changed graduation plans which resulted in a schedule change, Mike failed to accrue necessary credits for enrollment and was transferred to Joshua High School, and Jeff was court ordered to enroll in the High School Equivalency Program and begin General Education Development testing.

Conclusion

The purpose of this study was to implement a transition from traditional to reform-based instruction with at-risk learners and to examine how to best assist at-risk students in this transition, as well as changes in student attitudes, perception of their abilities, and performance. This transition within my classroom was, and is, an ongoing process. Determining whether direct answers to the research questions could be established was difficult; however, change was recorded in regard to students' attitudes and perception of their abilities. After coding the data,

the frequency of codes during each phase indicated a positive trend toward confidence and self-efficacy. Although I anticipated resistance and even hostility (Yates, 2009), students were willing to physically take part in the lessons even if they verbally expressed self-defeating statements. Students transitioned from frequent self-defeating statements and repetitive questions to gradually helping others and trusting other members in their group to answer their questions. The students' survey responses indicated positive attitudes. Most importantly, students acknowledged the difficulty of the tasks yet expressed confidence in their abilities. This displays perseverance, a trait that defies the characteristics of learned helplessness (Dweck, 1975; Diener & Dweck, 1978; Seifert, 2004). Slight improvement in test scores was noted with two students, but overall students who tended to perform well on tests at the beginning of the semester continued to perform well, and students who struggled with testing continued to struggle.

Through the implementation of the action research study in my classroom, I observed some unexpected surprises. I expected to hear off-task or inappropriate conversation while listening to the recordings; however, the students remained on task even if they struggled with the task. Several students asked repeated questions: a question asked once or more within the same lesson. Previously, I assumed that if students asked an identical question, they must not have been listening or paying attention to my previous answer. However, as I listened to the recordings, I noticed the students were not off task or talking during others' questions. I tried to determine the students' perception of my answers, and wondered why they continued to ask a question even if I had just answered an identical question. One conclusion could be that students saw me, the teacher, as the only valid source of information. Even if I had offered a response to another student, they seemed to have a need for a personal explanation in order to feel confident

enough to proceed. As the study continued, this need for repeated questions and answers not only began to decrease, but the students began questioning each other.

Table 5

Third Student Survey Responses

| Answer Choice | Percentage of Student Responses (n=4) |
|--|--|
| <i>When I think about Math now...</i> | |
| I think it is awesome. | 0% |
| I am doing a lot more than I thought I could! | 100% |
| I'm working hard and I've accomplished a lot. It's still hard. | 0% |
| I don't really like it, but it's not as bad as it was. | 0% |
| Impossible. I still hate it. Nothing has changed. | 0% |
| <i>Now I think Geometry is.....</i> | |
| An easy class. No problems. | 0% |
| The problems are hard but I can do it. | 100% |
| I'm still struggling but I'm passing. | 0% |
| Not my favorite class. It is hard and sometimes I don't do well on tests. | 0% |
| Impossible. I don't get it at all. Geometry is worse than I thought it would be | 0% |

Analyzing the students' motives and behavior required me to examine my teaching and my classroom from their perspective. As I listened to the audio recordings, I realized that I was providing more process help than I anticipated. I expected difficulties teaching on-level concepts

and remediation concurrently (Carnine, 1997), and the lab activities were deliberately selected to focus on important supporting skills. I did not plan to encourage dependence on me as the teacher (Hiebert et al., 1997); however, a substantial amount of scaffolding was necessary due to their extreme lack of prior knowledge and low level of basic supporting skills. I quickly found that the majority of lab time was spent in review and practice of basic skills. The more challenging aspects of each lab activity were typically found at the conclusion, but frequently the class period ended prior to the completion of a lab. Although I made an effort to encourage students without jumping in too quickly to answer their questions, I continued to find that I was helping more than I realized. As I studied the data through the cycle of action research, I was surprised by the extent of instructional time spent addressing lack of prior knowledge. In the future, I would like to focus on developing activities, which address the lack of prior knowledge yet challenge students. Ideally, activities should review and support basic skills and build new tasks, which require higher level thinking.

The overall process of action research in the classroom was, at times, overwhelming. Anticipated time to reflect and record field notes was frequently dedicated to after-school tutoring or detention duty. Meeting the daily requirements of teaching while conducting rigorous research was difficult. Normal occurrences, such as routine emergency drills, absences, short schedules, and testing, created difficulties that required flexibility and adjustment. As I was examining data, especially through the coding process, I struggled with developing codes, which emerged from the data. I found myself attempting to fit my data into preconceived codes, and avoiding this pitfall required reflection and adjustment. I reviewed my list of codes often, and when adjustments were made I re-evaluated previously coded data. Coding data became a fluid process as more data were analyzed – codes were created, merged or deleted.

Action research is a daunting task for practicing teachers, and the data and process used in this study might be relevant to any teacher considering engaging in a study. Conducting an action research study, such as the present study in a classroom with at-risk students, can be very difficult, but quite rewarding. Through the rigorous cycle of action research, educators can gain insight into their students' perceptions and attitudes. By analyzing classroom interaction, teachers can adjust their instruction to meet the unique needs of at-risk students. Potentially, teachers could help at-risk students become independent thinkers and take responsibility for their own learning. Traditional student-teacher interaction could be transitioned to a more desired classroom model, which promotes reflection, communication, and conceptual understanding (NCTM, 2000).

Looking forward to future research, I would like to further analyze my students' attitudes and written work to determine if they would be able to maintain their confidence and skills in the next mathematics course. Would students be able to apply their skills at the next level? Would any of their learning habits – seeking help from others, attempting new problems, and feeling confident in their abilities even if they believe the content is difficult – transfer to other subjects such as English, Science or Social Studies? If students were able to successfully transfer their attitudes and skills, a change could take place in the culture of the entire campus. Independent learning skills could lead to better understanding of content in all areas and potentially students who were better prepared for college and/or the work force.

This action research study served to inform and improve my practice and is most relevant to my classroom; however, the findings may apply to any teacher experiencing the same difficulties with at-risk students. I learned about my perception of my own teaching and how it differs from my actual instruction, and I gained insight into the students' perspective. Through

this research, I identified areas to improve in my own practice including a need to develop lessons to build supporting skills while incorporating high level thinking tasks, and a plan to bring my students' up to grade level more efficiently.

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Appendix A

Initial Survey Questions

Math is.....

- My favorite subject.
- Ok, I don't really have any problems with it.
- Very Hard. I don't like Math.
- Impossible. I hate it.

I think Geometry will be.....

- Super easy.
- Challenging, but I think I can handle it.
- Hard. I'm worried about passing.
- Impossible. I will fail for sure.

I took Geometry before. I didn't pass last year because.....

- I didn't do my homework and I got zeros.
- I failed my tests, they were really hard.
- The teacher wouldn't help me. Nothing made sense.
- I just don't get it at all.

The hardest part about math is:

Describe a typical math class. What happens? Is it always the same? Or different some days?

Appendix B

Second Survey Questions

When I think about Math now.....

- I still love it.
- It's getting better. I feel more confident.
- It's still hard, but I think I will pass.
- Impossible. I still hate it. Nothing has changed.

I think Geometry is.....

- An easy class. No problems.
- It is a little hard, but I'm getting it.
- It's really hard. I'm struggling.
- Impossible. I don't get it at all.

In Geometry, I am good at.....

Tell me about Geometry class. What do you like? What do you not like?:

Appendix C

Third Survey Questions

When I think about Math now.....

- I think it is awesome.
- I am doing a lot more than I thought I could!
- I'm working hard and I've accomplished a lot. It's still hard.
- I don't really like it, but it's not as bad as it was.
- Impossible. I still hate it. Nothing has changed.

Now I think Geometry is.....

- An easy class. No problems.
- The problems are hard but I can do it.
- I'm still struggling, but I'm passing.
- Not my favorite class. It is hard and sometimes I don't do well on tests.
- Impossible. I don't get it at all. Geometry is worse than I thought it would be.

In Geometry, something I need to work on or don't understand is.....

Tell me about your favorite Geometry class. Or, tell me about one that you hated.

Appendix D

Timeline of Units and Labs

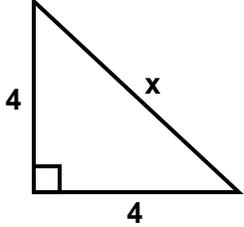
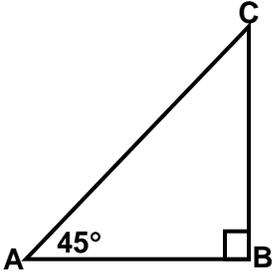
| Unit | Topic | Lab | Source |
|----------------|--------------------------|--------------------------------------|--|
| <i>Phase 1</i> | | | |
| One | Triangles | Pythagorean Theorem | |
| Two | Distance and Measurement | As the Crow Flies | Martin, W. <i>As the crow flies</i> . (2011). Retrieved from http://www.nctm.org/uploadedFiles/Journals_and_books/Books/FHSM/RSM_Task/crow.pdf . |
| | Measurement and Ratios | Human Ratios vs. Golden Ratio | |
| <i>Phase 2</i> | | | |
| Three | Area and Perimeter | Finding the Area of Irregular Shapes | Johnston, C. <i>Finding the area of irregular figures</i> . Retrieved from http://illuminations.nctm.org/Lesson.aspx?id=1902 |
| Four | Circles | Apple Pi | Johnston, C. <i>The ratio of circumference to diameter</i> . Retrieved from http://illuminations.nctm.org/Lesson.aspx?id=1849 . |

Phase 3

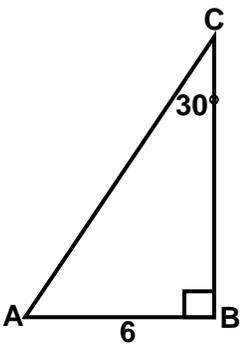
| | | | |
|------|-------------------------------------|---------------------------------|---|
| Five | Cylinders, Cones, and Spheres | Popcorn Prisms, Anyone? | Chaikin, J. <i>Popcorn, anyone?</i> Retrieved from http:// illuminations.nctm.org/Lesson. aspx?id=2927 |
| Six | Logic | Conditional Statements Story | |

Appendix E

Benchmark Test

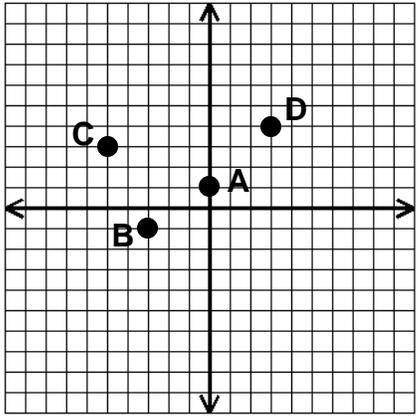
| | |
|-----------------|--|
| <p>_____ 1.</p> | <p>Using the triangle at right, find the value of 'x'.</p> <p>A. 4</p> <p>B. $4\sqrt{2}$</p> <p>C. 8</p>  |
| <p>_____ 2.</p> | <p>In the figure at right, $BC = 10$. Find AC.</p> <p>A. $5\sqrt{2}$</p> <p>B. $10\sqrt{2}$</p> <p>C. $8\sqrt{3}$</p> <p>D. None of these</p>  |

| | |
|----------------|--|
| <p>_____3.</p> | <p>In the figure at right, find AC.</p> <p style="margin-left: 40px;">A. 3</p> <p style="margin-left: 40px;">B. 12</p> <p style="margin-left: 40px;">C. $3\sqrt{3}$</p> <p style="margin-left: 40px;">D. $6\sqrt{3}$</p> |
|----------------|--|



Refer to the coordinate plane below to find each measure.

| |
|--|
| <p>4. BA = _____</p> <p style="margin-top: 20px;">B(-3, -1) A(0, 1)</p> |
|--|



Given three points **A**, **B**, and **M**, with **M** as the midpoint of \overline{AB} , find each of the following.

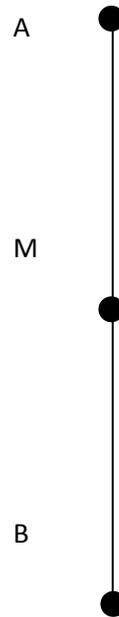
5. $A(-9, 4)$ and $B(3, -2)$

Find the coordinates of **M**.

$M(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

6. $A(-5, -4)$ and $M(1, -5)$

Find the coordinates of **B**.



$B(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

7. _____

If a 14-foot tree casts a 6-foot-long shadow. How tall is a tree that casts a 4-foot-long shadow?

Quadrilateral ABCD is similar to EFGH. Identify the corresponding parts.

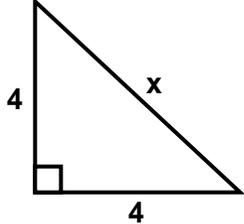
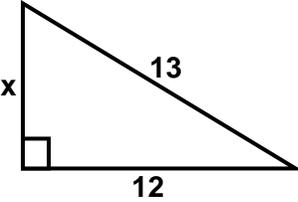
| | |
|------------|--|
| 8. | $\angle A \cong \angle \underline{\hspace{2cm}}$ |
| 9. | $\angle \underline{\hspace{2cm}} \cong \angle G$ |
| 10. | $\angle D \cong \angle \underline{\hspace{2cm}}$ |
| 11. | $\angle \underline{\hspace{2cm}} \cong \angle F$ |

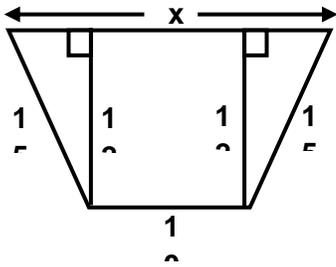
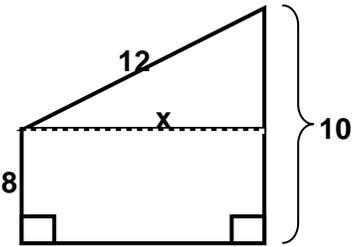
Appendix F

Unit One Test

PART 1. PYTHAGOREAN THEOREM

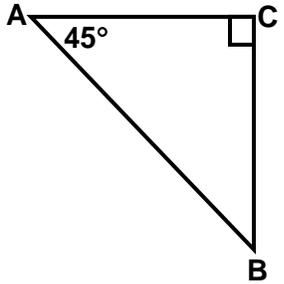
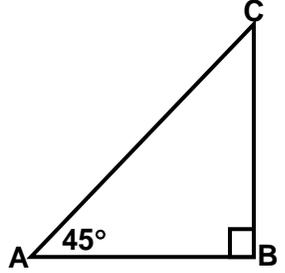
Choose the correct answer for each of the following, writing its corresponding letter in the blank provided.

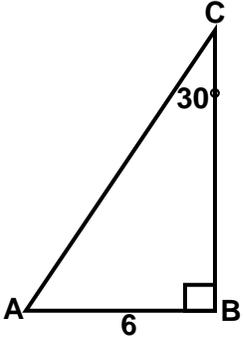
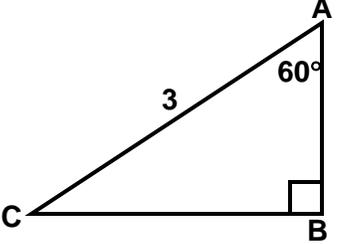
| | |
|----------|---|
| _____ 1. | <p>Using the triangle at right, find the value of 'x'.</p> <p>D. 4</p> <p>E. $4\sqrt{2}$</p> <p>F. 8</p> <p>G. 32</p> <p>H. None of these.</p>  |
| _____ 2. | <p>Using the triangle at right, find the value of 'x'.</p> <p>A. 5</p> <p>B. 1</p> <p>C. 25</p> <p>D. 7</p> <p>E. 12</p>  |

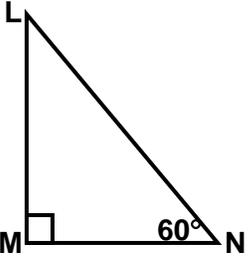
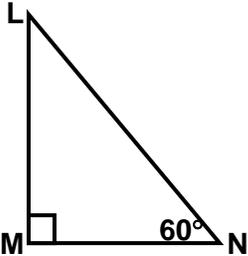
| | |
|---------------------------|--|
| <p>_____3.</p> | <p>A rectangle has a width of 9 cm, and a diagonal of 15 cm. Find the length.</p> <p>A. 12 cm</p> <p>B. 6 cm</p> <p>C. $2\sqrt{3}$ cm</p> <p>D. 9 cm</p> <p>E. None of these.</p> |
| <p><u> X = </u> 4.</p> | <p>Find X. Show all work.</p>  |
| <p><u> X = </u> 5.</p> | <p>Solve for X. Show all work.</p>  |

PART 3. 45°-45°-90° AND 30°-60°-90° TRIANGLES

Choose the correct answer for each of the following, writing its corresponding letter in the blank provided.

| | |
|-----------------|--|
| <p>_____ 6.</p> | <p>In the triangle at right, $AC = 24$. Find AB.</p> <p>A. 12 B. 24 C. $12\sqrt{2}$ D. $24\sqrt{2}$ A. None of these</p>  |
| <p>_____ 7.</p> | <p>In the figure at right, $BC = 10$. Find AC.</p> <p>D. 10 E. $5\sqrt{2}$ F. $10\sqrt{2}$ G. $8\sqrt{3}$ E. None of these.</p>  |

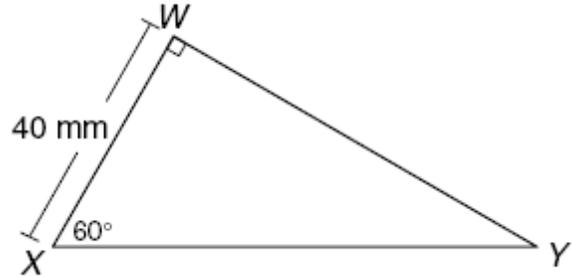
| | |
|----------|---|
| _____ 8. | <p>In the figure at right, find AC.</p> <p>E. 3</p> <p>F. 12</p> <p>G. $3\sqrt{3}$</p> <p>H. $6\sqrt{3}$</p> <p>H. None of these.</p>  <p>The diagram shows a right-angled triangle ABC. The right angle is at vertex B. The angle at vertex C is labeled as 30°. The side opposite to angle C, which is side AB, is labeled with the length 6. The hypotenuse is side AC.</p> |
| _____ 9. | <p>Find the value of BC in the figure at right.</p> <p>A. $32\sqrt{3}$</p> <p>B. 16</p> <p>C. 8</p> <p>D. $8\sqrt{3}$</p> <p>E. $16\sqrt{3}$</p>  <p>The diagram shows a right-angled triangle ABC. The right angle is at vertex B. The angle at vertex A is labeled as 60°. The hypotenuse, which is side AC, is labeled with the length 3. The side opposite to angle A, which is side BC, is the unknown side to be found.</p> |

| | | |
|-----------------|--|---|
| <p>_____10.</p> | <p>In the triangle at right, $MN = 4$. What is the length of LM?</p> <p>A. 4 B. 8 C. $4\sqrt{3}$ D. $4\sqrt{6}$ E. None of these.</p> |  |
| <p>_____11.</p> | <p>In the triangle at right, $LM = 6\sqrt{3}$. What is the length of LN?</p> <p>A. 12 B. 6 C. $6\sqrt{6}$ D. $3\sqrt{6}$ E. None of these.</p> |  |

12. $\triangle WXY$ is a right triangle.

Find the length of \overline{WY} .

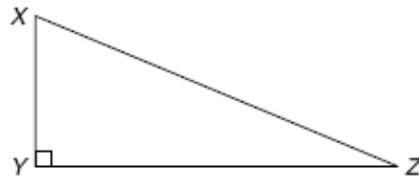
- A 20 mm
- B $20\sqrt{3}$ mm
- C 60 mm
- D $40\sqrt{3}$ mm



BONUS

If $XY = 8$ feet and $XZ = 17$ feet, what is the area of $\triangle XYZ$?

- F 15 ft^2
- G 30 ft^2
- H 60 ft^2
- J 120 ft^2



Appendix G

Unit Two Test

Find the value of 'x' and the length of the segment indicated.

1. If B is between A and C, and $AB = 3x + 1$ and $BC = 2x - 7$, and $AC = 24$, then find the value of 'x' and AB.

$$x = \underline{\hspace{2cm}}$$

2. What is the value AB in problem #1.

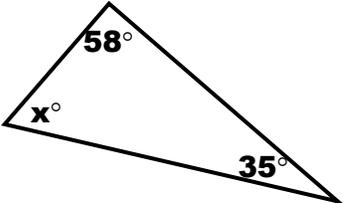
$$AB = \underline{\hspace{2cm}}$$

Match each term with its appropriate description. All descriptions will be used, but they will not repeat.

| | |
|--------------------------------|--|
| _____ 7. Isosceles Triangle | A. a closed figure formed by three or more segments that intersect each other at their endpoints |
| _____ 8. Acute Triangle | B. a triangle with no two sides congruent |
| _____ 9. Equiangular Triangle | C. a triangle with at least two sides congruent |
| _____ 10. Obtuse Triangle | D. a triangle with all sides congruent |
| _____ 11. Scalene Triangle | E. a triangle all of whose angles are acute angles |
| _____ 12. Equilateral Triangle | F. a triangle with an obtuse angle |
| _____ 13. Triangle | G. a triangle with all angles congruent |
| _____ 14. Right Triangle | H. a triangle with one 90 degree angle. |

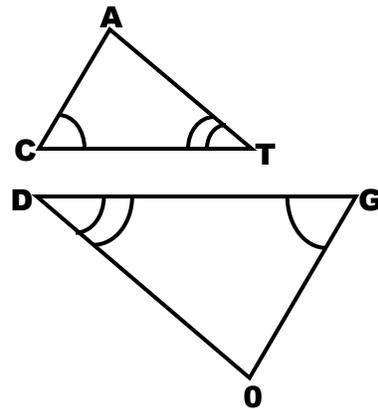
ANGLES OF TRIANGLES

Find the value 'x' in each problem, and write your answer in the blank provided.

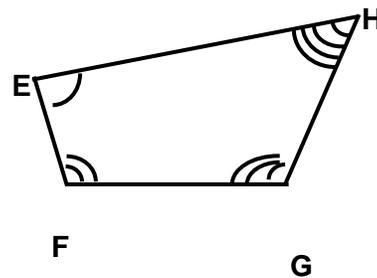
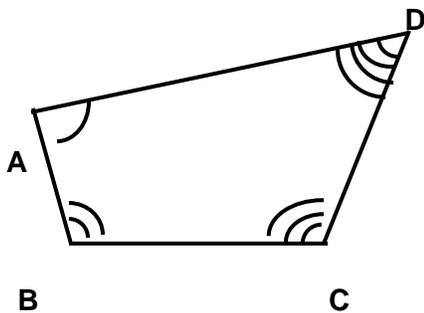
| | |
|-----------|---|
| _____ 15. |  |
|-----------|---|

Use the similar triangles below to determine whether each of the following statements are **TRUE** or **FALSE**. (Circle one.)

| | |
|-------------------|--|
| 16. TRUE or FALSE | $\angle C \cong \angle D$ |
| 17. TRUE or FALSE | $\angle O$ is included between \overline{DO} and \overline{GO} . |
| 18. TRUE or FALSE | $\overline{AT} \parallel \overline{OD}$ |
| 19. TRUE or FALSE | \overline{OD} is included between $\angle O$ and $\angle G$. |

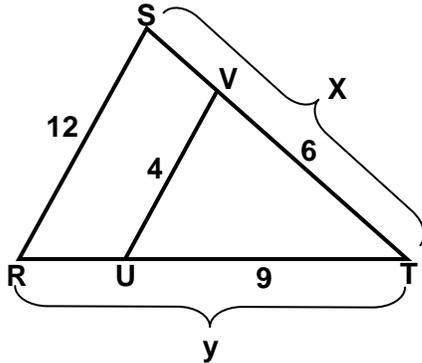


Quadrilateral ABCD is similar to EFGH. Identify the corresponding parts.

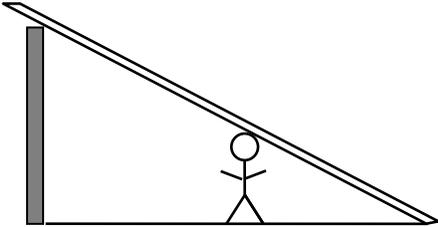


| | |
|-----|--|
| 20. | $\angle A \cong \angle \underline{\hspace{2cm}}$ |
| 21. | $\angle \underline{\hspace{2cm}} \cong \angle G$ |
| 22. | $\angle D \cong \angle \underline{\hspace{2cm}}$ |
| 23. | $\angle \underline{\hspace{2cm}} \cong \angle F$ |

Two similar polygons are shown. Find the values of 'x' and 'y'.

| | |
|--|---|
| <p>24. $x =$ _____</p> <p>$y =$ _____</p> | <p>$\triangle RST \sim \triangle UVT$</p>  |
|--|---|

Solve each of the following, and write your final answer in the blank provided.

| | |
|------------------|---|
| <p>25. _____</p> | <p>A ladder rests against the top of a wall. The head of a person 6 feet tall just touches the ladder. The person is 9 feet from the wall and 7 feet from the foot of the ladder. Find the height of the wall.</p>  |
|------------------|---|

26. _____

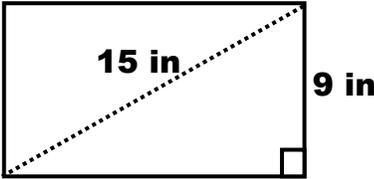
If a 14-foot tree casts a 6-foot-long shadow. How tall is a tree that casts a 4-foot-long shadow?

Appendix H

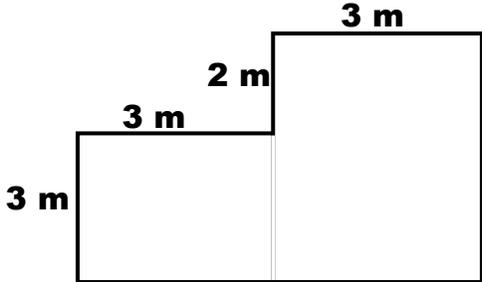
Unit Three Test

PART 1. AREA & PERIMETER OF RECTANGLES

Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

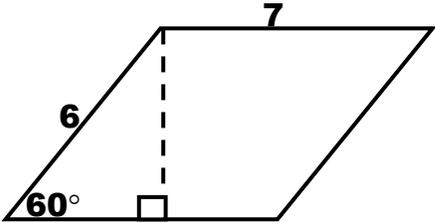
| | |
|----------|--|
| _____ 1. | <p>Find the area of a rectangle that is 12 cm long and 4 cm wide.</p> <p>A. 20 sq. cm</p> <p>B. 48 sq. cm</p> <p>C. 32 sq. cm</p> <p>D. 24 sq. cm</p> <p>E. 16 sq. cm</p> |
| _____ 2. | <p>Find the perimeter of the rectangle shown below.</p> <p>A. 42 in</p> <p>B. 21 in</p> <p>C. 108 in</p> <p>D. 135 in</p> <p>E. 48 in</p>  <p>The diagram shows a rectangle with a dotted diagonal line. The diagonal is labeled "15 in". The right vertical side is labeled "9 in". A small square at the bottom right corner indicates a right angle.</p> |

| | |
|---------|--|
| _____3. | <p>Find the area of a square with a perimeter of 60 m.</p> <p>A. 400 m^2</p> <p>B. 196 m^2</p> <p>C. 36 m^2</p> <p>D. 625 m^2</p> <p>E. 225 m^2</p> |
|---------|--|

| | |
|---------|---|
| _____4. | <p>What is the area of the figure below?</p>  <p>A. 16 m^2</p> <p>B. 29 m^2</p> <p>C. 14 m^2</p> <p>D. 24 m^2</p> <p>E. None of these.</p> |
|---------|---|

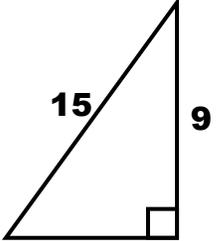
AREA & PERIMETER OF PARALLELOGRAMS

Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

| | |
|---------|--|
| _____5. | <p>The area of a parallelogram is 4725 square units, and its height is 75 units. Find the length of the base.</p> <p>A. 56 u. B. 63 u. C. 53 u. D. 52 u. E. 65 u.</p> |
| _____6. | <p>Find the area of the parallelogram below.</p> <p>A. $24\sqrt{3} \text{ u}^2$ B. 48 u^2 C. 24 u^2 D. $12\sqrt{3} \text{ u}^2$ E. None of these</p>  |

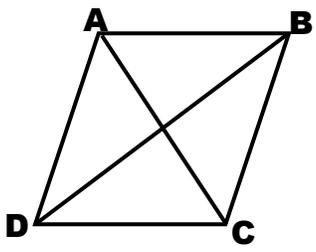
AREA & PERIMETER OF TRIANGLES

Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

| | |
|----------|--|
| _____ 7. | <p>Find the area of a triangle with a base of 16 m and a height of 7 m.</p> <p>A. 15 m^2</p> <p>B. 28 m^2</p> <p>C. 56 m^2</p> <p>D. 112 m^2</p> <p>E. 23 m^2</p> |
| _____ 8. | <p>Find the area of the triangle below.</p> <p>A. 6 u^2</p> <p>B. 30 u^2</p> <p>C. 24 u^2</p> <p>D. 54 u^2</p> <p>E. None of these.</p>  <p>The diagram shows a right-angled triangle. The vertical leg on the right is labeled with the number 9. The hypotenuse is labeled with the number 15. A small square at the bottom-right vertex indicates a right angle.</p> |

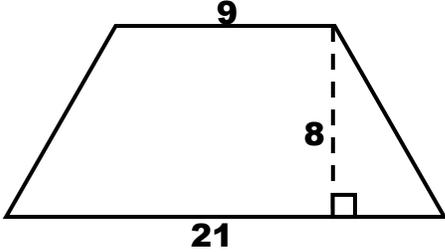
AREA & PERIMETER OF RHOMBI

Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

| | |
|----------|---|
| _____9. | <p>If the diagonals of a rhombus are 7 cm and 12 cm long, find the area of the rhombus.</p> <p>21 cm²</p> <p>A. 42 cm²</p> <p>B. 84 cm²</p> <p>C. 168 cm²</p> <p>D. None of these.</p> |
| _____10. | <p>Find the area of the rhombus below.</p> <p>A. 54 u²</p> <p>B. 27 u²</p> <p>C. 15 u²</p> <p>D. 9 u²</p> <p>A. None of these.</p> <div data-bbox="1006 997 1469 1249"><p>AC = 6 DB = 5</p></div> |

AREA & PERIMETER OF TRAPEZOIDS

Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

| | |
|-----------|--|
| _____ 11. | <p>A trapezoid with a height of 8 ft has bases of lengths 14 ft and 12 ft. Find the area of the trapezoid.</p> <p>A. 104 ft^2</p> <p>B. 52 ft^2</p> <p>C. 208 ft^2</p> <p>D. 1344 ft^2</p> <p>E. None of these.</p> |
| _____ 12. | <p>Find the area of the trapezoid below.</p> <p>A. 184 u^2</p> <p>B. 88 u^2</p> <p>C. 76 u^2</p> <p>D. 120 u^2</p> <p>E. 92 u^2</p>  |

Appendix I

Unit Four Test

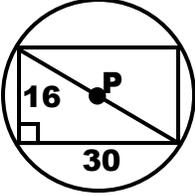
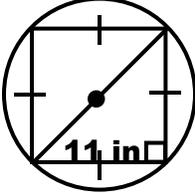
CIRCLE BASICS

Match each term with its appropriate description. All descriptions will be used, none will repeat.

| | |
|--------------------|--|
| _____ 1. Chord | A. A line that intersects a circle at exactly two points |
| _____ 2. Diameter | B. A line, ray, or segment that intersects a circle at exactly one point |
| _____ 3. Secant | C. A segment with both endpoints on the circle |
| _____ 4. Tangent | D. An arc with a measure less than 180° |
| _____ 5. Major Arc | E. A chord that goes through the center of a circle |
| _____ 6. Minor Arc | F. An arc with a measure more than 180° |

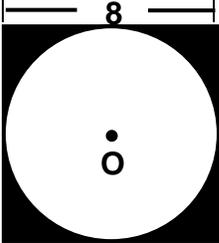
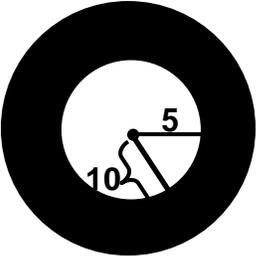
Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

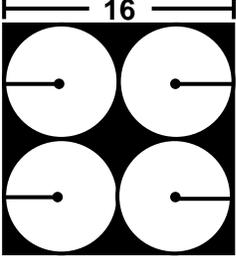
| | |
|---------|---|
| _____7. | <p>In a given circle, the radius is 41cm. The measure of this circle's diameter is:</p> <p>A. 82π cm</p> <p>B. 41π cm</p> <p>C. 82 cm</p> <p>D. 20.5 cm</p> |
| _____8. | <p>In a given circle, the diameter is 36 in. The measure of this circle's radius is:</p> <p>A. 18 in</p> <p>B. 72 in</p> <p>C. 18π in</p> <p>D. 72π in</p> |
| _____9. | <p>In a given circle, the diameter is 6 m. The circumference of the circle is</p> <p>A. 9.42 m</p> <p>B. 113.097 m</p> <p>C. 28.274 m</p> <p>D. 18.85 m</p> |

| | |
|------------------|---|
| <p>_____ 10.</p> | <p>Find the area of Circle P:</p>  <p>A. 17π square units B. 34π square units C. 289π square units D. 1156 square units</p> |
| <p>_____ 11.</p> | <p>Find the area of the circle:</p>  <p>A. 134.40 in^2 B. 190.07 in^2 C. 380.13 in^2 D. 760.27 in^2</p> |

AREAS OF SHADED REGIONS

Find the correct answer for each of the following. Write its corresponding letter in the blank provided.

| | |
|------------------|---|
| <p>_____ 12.</p> | <p>Find the EXACT area of the shaded region:</p> <div style="text-align: right;">  </div> <p>A. $64 - 16\pi \text{ cm}^2$</p> <p>B. $64 - 64\pi \text{ cm}^2$</p> <p>C. $64 - 8\pi \text{ cm}^2$</p> <p>D. 64 cm^2</p> |
| <p>_____ 13.</p> | <p>Find the area of the shaded region:</p> <div style="text-align: right;">  </div> <p>A. 31.416 square units</p> <p>B. 235.619 square units</p> <p>C. 392.699 square units</p> <p>D. 942.478 square units</p> |

| | |
|-----------|---|
| _____ 14. | <p>Find the EXACT area of the shaded region</p> <p>A. $256 - 16\pi$ square units</p> <p>B. $256 - 64\pi$ square units</p> <p>C. $64 - 64\pi$ square units</p> <p>D. $256 - 256\pi$ square units</p>  |
|-----------|---|

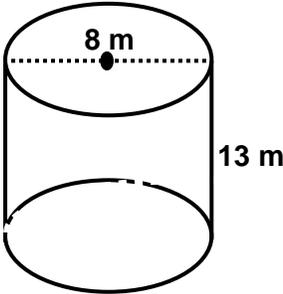
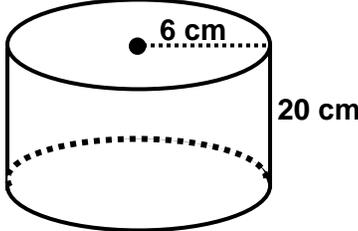
Appendix J

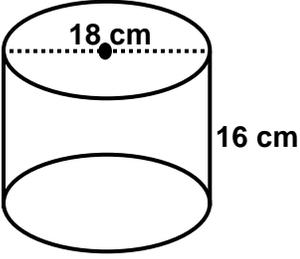
Unit Five Test

AREA & VOLUME OF CYLINDERS, CONES, & SPHERES

CYLINDERS

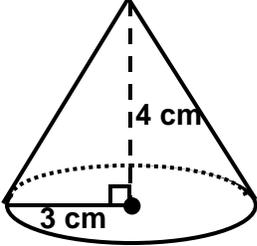
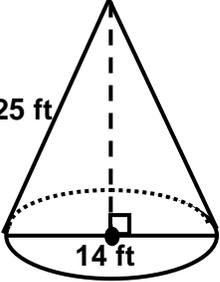
Find the correct answer for each of the following, writing its corresponding letter in the blank provided.

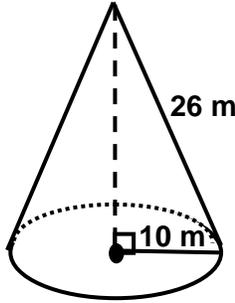
| | |
|-----------------|--|
| <p>_____ 1.</p> | <p>Find the Lateral Area of the cylinder:</p>  <p>A. $208\pi \text{ m}^2$ B. $120\pi \text{ m}^2$ C. $104\pi \text{ m}^2$ D. $8\pi \text{ m}^2$</p> |
| <p>_____ 2.</p> | <p>Find the Total Area of the cylinder:</p>  <p>A. 226 cm^2 B. 754 cm^2 C. 980 cm^2 D. 2262 cm^2</p> |

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| _____3. | <p>Find the Volume of the cylinder:</p>  <p>A. $81\pi \text{ cm}^3$ B. $288\pi \text{ cm}^3$ C. $450\pi \text{ cm}^3$ D. $1296\pi \text{ cm}^3$</p> |
| _____4. | <p>The Lateral Area of a right circular cylinder is 48π square meters. The height is 6 m.</p> <p>Find the radius of the base.</p> <p>A. 4 m B. 8 m C. 6 m D. 12 m</p> |
| _____5. | <p>The Total area of a cylinder is 96π square inches. If the radius is 4 inches long, find the height.</p> <p>A. 32 in B. 100.5 in C. 8 in D. 25.1 in</p> |

CONES

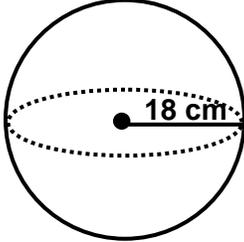
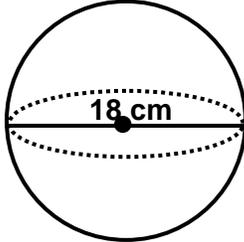
Find the correct answer for each of the following, writing the correct answer in the blank provided.

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| <p>_____6.</p> | <p>Find the Lateral Area of the right circular cone:</p> <p>A. 47.124 cm^2</p> <p>B. 75.398 cm^2</p> <p>C. 15.708 cm^2</p> <p>D. 37.699 cm^2</p>  <p>The diagram shows a right circular cone. A vertical dashed line from the apex to the center of the base represents the height, labeled '4 cm'. A horizontal line from the center of the base to the edge represents the radius, labeled '3 cm'. A right-angle symbol is shown at the center of the base where the height and radius meet.</p> |
| <p>_____7.</p> | <p>Find the Total Area of the right circular cone: 25 ft</p> <p>A. 75.4 ft^2</p> <p>B. 703.7 ft^2</p> <p>C. 241.9 ft^2</p> <p>D. 549.8 ft^2</p>  <p>The diagram shows a right circular cone. A vertical dashed line from the apex to the center of the base represents the height. A horizontal line from the center of the base to the edge represents the radius, labeled '14 ft'. A slant height is labeled '25 ft'. A right-angle symbol is shown at the center of the base where the height and radius meet.</p> |

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| _____ 8. | <p>Find the Volume of the right circular cone:</p> <p>A. $\frac{800\pi}{3} \text{ m}^3$</p> <p>B. $800\pi \text{ m}^3$</p> <p>C. $920\pi \text{ m}^3$</p> <p>D. $520\pi \text{ m}^3$</p>  <p>The diagram shows a right circular cone. A vertical dashed line represents the height, extending from the apex to the center of the circular base. A solid horizontal line represents the radius, extending from the center of the base to the edge. A right-angle symbol is shown at the intersection of the height and the radius. The slant height, which is the length of the side of the cone, is labeled as 26 m. The radius is labeled as 10 m.</p> |
| _____ 9. | <p>The Lateral Area of a right circular cone is 65π. Its slant height is 13. Find the length of the diameter of the base.</p> <p>A. 5 units</p> <p>B. 5.42 units</p> <p>C. 10 units</p> <p>D. 10.83 units</p> |
| _____ 10. | <p>The Volume of a right circular cone is 9π cubic centimeters and its height is 3 cm.</p> <p>Find the length of its radius.</p> <p>A. 27 cm</p> <p>B. 9 cm</p> <p>C. 3 cm</p> <p>D. $3\sqrt{3}$ cm</p> |

SPHERES

Find the correct answer for each of the following, writing the correct answer in the blank provided.

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| _____ 11. | <p>Find the Total Area of the sphere:</p> <p>A. $23328\pi \text{ cm}^2$</p> <p>B. $7776\pi \text{ cm}^2$</p> <p>C. $1296\pi \text{ cm}^2$</p> <p>D. $324\pi \text{ cm}^2$</p>  |
| _____ 12. | <p>Find the Volume of the sphere:</p> <p>A. 254.5 cm^3</p> <p>B. 1017.9 cm^3</p> <p>C. 3053.6 cm^3</p> <p>D. $24,429.0 \text{ cm}^3$</p>  |
| _____ 13. | <p>The Total Area of a sphere is $144\pi \text{ cm}^2$. Find its radius.</p> <p>A. 6 cm</p> <p>B. 12 cm</p> <p>C. 18 cm</p> <p>D. 36 cm</p> |

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| _____14. | <p>The Volume of a sphere is $36\pi \text{ cm}^3$. Find its radius.</p> <p>A. 108 cm</p> <p>B. 36 cm</p> <p>C. 6 cm</p> <p>D. 3 cm</p> |
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Appendix K

Unit Six Test

CONDITIONAL STATEMENTS & BEGINNING PROOFS**CONDITIONAL STATEMENTS**

Find the correct answer for each of the following, and write its corresponding letter in the blank provided.

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| ____ 1. | <p>In the conditional statement <i>“If an angle is acute, then it has a measure less than 90°.”</i> the conclusion is:</p> <p>A. if an angle is acute</p> <p>B. an angle is acute</p> <p>C. then it has a measure less than 90°</p> <p>D. it has a measure less than 90°</p> |
| ____ 2. | <p>In the conditional statement, <i>“If you give a mouse a cookie, then he will want some milk to go with it.”</i> the hypothesis is:</p> <p>A. if you give a mouse a cookie</p> <p>B. you give a mouse a cookie</p> <p>C. then he will want some milk to go with it</p> <p>D. he will want some milk to go with it</p> |

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| _____3. | <p>Which of the following is the converse of the given conditional?</p> <p><i>If you break an object in the store, then you must pay for it.</i></p> <p>A. If you have to pay for an object in a store, then you must have broken it.</p> <p>B. If you do not break an object in the store, then you don't have to pay for it.</p> <p>C. If you do not have to pay for an object, then you must not have broken it.</p> <p>D. You must pay for an object if and only if you have broken it.</p> |
| _____4. | <p>Which of the following is the inverse of the given conditional?</p> <p><i>If you spill your drink, then the carpet will get stained.</i></p> <p>A. If you do not spill your drink, then the carpet will not get stained.</p> <p>B. If the carpet is stained, then you must have spilled your drink.</p> <p>C. If the carpet is not stained, then you didn't spill your drink.</p> <p>D. The carpet will get stained if and only if you spill your drink.</p> |
| _____5. | <p>Which of the following is the contrapositive of the given conditional?</p> <p><i>If you want to run in a marathon, then you must train hard.</i></p> <p>A. You must train hard if and only if you want to run in a marathon.</p> <p>B. If you do not want to run in a marathon, then you do not have to train hard.</p> <p>C. If you are training hard, then you must want to run in a marathon.</p> <p>D. If you are not training hard, then you must not want to run in a marathon.</p> |

ALGEBRA PROPERTIES

Match each conditional statement with the algebraic property that would be used to prove the conclusion. **NOTE: Not all answer choices will be used, nor will they be repeated.**

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| _____ 6. If $3x = 180$, then $x = 60$ | A. Symmetric Property |
| _____ 7. If $j = k$ and $k = m$, then $j = m$ | B. Transitive Property |
| _____ 8. If a , then $a = a$. | C. Multiplication Property |
| _____ 9. If $3x - 2 = 7$, then $3x = 9$. | D. Reflexive Property |
| _____ 10. If $AB = CD$, then $CD = AB$. | E. Addition Property |
| | F. Division Property |
| | D. Substitution Property |

TWO-COLUMN PROOFS

Using the given statements and reasons, complete the following two-column proofs. You need only write in the corresponding letters in your proof. NOTE: Not all statements and reasons will be used in each proof, nor will they be repeated.

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| 11. GIVEN: $4(x + 3) = 36$ | | <ul style="list-style-type: none"> A. subtraction property B. $x = 6$ C. multiplication property D. Statements E. distributive property F. $4x = 24$ G. Given H. Reasons I. $4(x + 3) = 36$ J. prove K. division property L. $4x + 12 = 36$ M. addition property |
| PROVE: $x = 6$ | | |
| 11a) | 11b) | |
| 11c) | 11d) | |
| 11e) | 11f) | |
| 11g) | 11h) | |