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# Tachyon Behavior Due to Mass-State Transitions at Scattering Vertices

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**Abstract:** A particle beam-thin foil scattering model is updated within the context of parametrized relativistic quantum theory (pRQT). This paper focuses on the creation, annihilation, and detection of tachyons when a beam of particles scatters off a thin foil. Improved calculation procedures and recent data are used to update model calculations for a pion-proton system.

**Keywords:** accelerator; parametrized relativistic quantum theory (pRQT); tachyon; mass state transition

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## 1. Introduction

Mass state transitions at scattering vertices can be used to test parametrized relativistic quantum theory (pRQT), a manifestly covariant quantum theory with an invariant evolution parameter. Introductions to pRQT are presented by myself [1,2], Pavšič [3,4], and Horwitz [5]. A review of relativistic classical mechanics and electrodynamics in the parametrized framework is given by Land and Horwitz [6]. In [7], I developed a parametrized relativistic quantum field theory that was extended by Pavšič (see [3], Chapter 1) to include canonical quantization and creation/annihilation operators. Horwitz ([5], Chapter 3) discussed Fock space and quantum field theory. Additional topics include branes and quantized fields [4,8]. I [1,2] and Pavšič [3,4,9] discussed the role of action-at-a-distance, nonlocality, and tachyons in pRQT.

Transitions from one mass state to another are induced in pRQT when a system with discrete mass states interacts with a parameter-dependent perturbation [10,11]. This is analogous to the transition from one energy state to another when a system with discrete energy states interacts with a time-dependent perturbation in Schroedinger quantum mechanics.

Tachyon physics in pRQT differs from other formulations [1,9,12]. Pavšič [4] clarified and analyzed tachyon topics that he identified as misconceptions and confusing. For example, the conventional view is that tachyons have imaginary mass. By contrast, as shown below, tachyons in pRQT have real mass. Furthermore, mass-state transitions in pRQT provide a mechanism for creating, annihilating, or detecting tachyons.

The purpose of this paper is to highlight tachyon creation and annihilation cases that could facilitate the design of experimental tests for pRQT. The cases were introduced in a previous publication [12] where they were a relatively small part of a more comprehensive study. Recent experimental results [13] show that results reported previously [12] should be updated. Some background material and new calculation procedures are included to help the reader understand and apply the procedures. References are provided for readers who would like more information.

Particle mass in parametrized relativistic classical mechanics (pRCM) and pRQT is discussed in Sections 2 and 3, respectively. It is shown that tachyon mass is real and positive in both pRCM (Section 2) and pRQT (Section 3). The theory of mass state transitions in pRQT and a model of particle scattering off a thin foil [12] are reviewed in Section 4. Section 5 contains updated results for calculating tachyon creation, annihilation, and detection. The existence of alternative theories of tachyons is recognized and the opportunity to test pRQT tachyon physics is highlighted in Section 6. Details of the calculations are succinctly outlined in two new appendices and are more direct than procedures reported previously [12].

## 2. Parametrized Relativistic Classical Mechanics and Classical Mass

The meaning of the concept of mass in pRQT differs from the conventional view of mass. According to the conventional view, mass is a property of a particle that must be entered into calculations. The difference between the conventional view of mass and the pRQT view is demonstrated by first considering the concept of mass in pRCM.

### *2.1. Lagrangian and Hamiltonian Formulations of pRCM*

The relationship between mass and motion of a free particle illustrates the role of mass in pRCM. We begin the evaluation of mass and motion in pRCM by outlining parametrized Lagrangian and Hamiltonian formulations. This leads to formulations that are applicable to a parametrized Hamiltonian for a classical free particle.

Hamilton's variational principle is used to develop pRCM by first defining the action,

$$A = \int_{s_1}^{s_2} L_S(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n, s) ds, \tag{1}$$

with invariant evolution parameter $s$. The Lagrangian, $L_S$, is a function of $n$ generalized coordinates $\{q_1, \ldots, q_n\}$. The generalized coordinate, $\dot{q}_i$, denotes differentiation of $q_i$ with respect to $s$. The subscript $S$ in $L_S$ indicates that $L_S$ is a Lagrangian that satisfies a special relativistic metric. The evolution parameter $s$ is a relativistic scalar that does not depend on generalized coordinates. Parameter end points, $s_1$ and $s_2$, are fixed.

The functional form of $L_S$ is

$$L_S = L_S\left(q^\mu, \dot{q}^\mu, s\right) = L_S\left(q^0, q^1, q^2, q^3, \dot{q}^0, \dot{q}^1, \dot{q}^2, \dot{q}^3, s\right), \tag{2}$$

where the generalized coordinates are elements of a spacetime four-vector, nonzero elements of the fundamental metric tensor $g_{\mu v}$, $\mu, v = 0(\text{time}), 1,\ 2,\ 3(\text{space})$, are $g_{00} = 1 = -g_{11} = -g_{22} = -g_{33}$, and

$$\dot{q} = \frac{dq}{ds}. \tag{3}$$

According to Hamilton's principle, the variation $\delta A$ of action $A$ must vanish:

$$\delta A = 0 = \int_{s_1}^{s_2} \delta L_S\left(q^\mu, \dot{q}^\mu, s\right) ds. \tag{4}$$

The solution of Equation (4) leads to Euler–Lagrange equations for a parametrized Lagrangian $L_S$:

$$\frac{\partial L_S}{\partial q^\mu} - \frac{d}{ds}\frac{\partial L_S}{\partial \dot{q}^\mu} = 0. \tag{5}$$

The Lagrangian formulation can be transformed into the Hamiltonian formulation by deriving canonical equations from the Legendre transformation:

$$\begin{aligned} K(q^\mu, p_\mu, s) &= \dot{q}^\mu p_\mu - L_S\left(q^\mu, \dot{q}^\mu, s\right) \\ &= K\left(q^0, q^1, q^2, q^3, p_0, p_1, p_2, p_3, .s\right). \end{aligned} \tag{6}$$

The function $K$ is a parametrized Hamiltonian function $K(q^\mu, p_\mu, s)$. The Legendre transformation can be used to derive Hamilton's equations for a parametrized Hamiltonian:

$$\dot{p}_\mu = -\frac{\partial K}{\partial q^\mu} \tag{7}$$

and

$$\dot{q}^{\mu} = \frac{\partial K}{\partial p_{\mu}} \tag{8}$$

## 2.2. Mass of a Classical Free Particle

The relationship between the mass and motion of a classical free particle is obtained by specifying a parametrized Hamiltonian. A free particle with constant mass, $M$, has the parametrized Hamiltonian,

$$K_f = \frac{g^{\mu v}}{2M} p_{\mu} p_{v}. \tag{9}$$

The spacetime four-vector, $q^{\mu,}$ and momentum–energy four-vector, $p_{\mu}$, are

$$\begin{aligned} q^{\mu} &= \left(q^0, q^1, q^2, q^3\right) = \left(ct, \vec{q}\right), \\ p_{\mu} &= \left(p_0, p_1, p_2, p_3\right) = \left(E/c, -\vec{p}\right), \end{aligned} \tag{10}$$

with the speed of light in vacuum, $c$, the time, $t$, and the energy, $E$. Substituting $K_f$ into Equations (7) and (8) gives:

$$\dot{q}^{\mu} = p^{\mu}/M \tag{11}$$

and

$$\dot{p}_{\mu} = 0. \tag{12}$$

The Lagrangian $L_{f,}$ corresponding to $K_{,f}$ is

$$L_f = \frac{g^{\mu v}}{2M}\left(M\dot{q}_{\mu}\right)\left(M\dot{q}_{v}\right) = \frac{M}{2}\dot{q}^{v}\dot{q}_{v}. \tag{13}$$

Equation (12) shows that the four-momentum of a free particle is constant:

$$p_{\mu} = p_{0\mu}. \tag{14}$$

Equation (11) is solved by substituting Equation (14) into Equation (11) and integrating over $s$ to find:

$$q^{\mu} = q_0^{\mu} + \frac{p_0^{\mu} s}{M} \text{ or } q^{\mu} - q_0^{\mu} = \frac{p_0^{\mu} s}{M}. \tag{15}$$

The terms $p_0^{\mu}, q_0^{\mu}$ are constants. According to Equation (15), the trajectory of a free particle is linearly dependent on $s$. If one varies the spacetime four-vector and form its inner product, then:

$$\delta q^{\mu} \delta q_{\mu} = \frac{p_0^{\mu} p_{0\mu}}{M^2} (\delta s)^2. \tag{16}$$

The term $M^2$ is found by rearranging Equation (16); thus,

$$M^2 = \frac{p_0^{\mu} p_{0\mu}}{\delta q^{\mu} \delta q_{\mu}} (\delta s)^2. \tag{17}$$

The invariant evolution parameter increases monotonically so that $\delta s > 0$. The terms $p_0^{\mu} p_{0\mu}$, $\delta q^{\mu} \delta q_{\mu}$ can be either timelike $p_0^{\mu} p_{0\mu} > 0$, $\delta q^{\mu} \delta q_{\mu} > 0$ or spacelike $p_0^{\mu} p_{0\mu} < 0$, $\delta q^{\mu} \delta q_{\mu} < 0$ for independent space–time coordinates and energy–momentum components. Consequently, $M^2$ is positive for both timelike and spacelike motion because negative signs associated with spacelike motion cancel. Free tachyons in pRCM have real mass.

## 3. Parametrized Relativistic Quantum Theory and the Meaning of Mass

The pRCM concepts, outlined in Section 2, are extended to the quantum case here. The pRQT field equation is the Stueckelberg equation for a single particle. It has the form:

$$i\hbar\frac{\partial\Psi}{\partial s} = K\Psi, \tag{18}$$

with mass operator

$$K = \frac{\pi^\mu\pi_\mu}{2m} + V, \tag{19}$$

and potential energy $V$. Here $\Psi$ is particle wave function and $\hbar$ is the reduced Planck constant.

The four-vector potential, $A^\mu$, is contained in the four-momentum operator $\pi^\mu$ with minimal coupling:

$$\pi^\mu = \frac{\hbar}{i}\frac{\partial}{\partial x_\mu} - \frac{e}{c}A^\mu. \tag{20}$$

The expectation value of an observable $\Omega$ in pRQT is

$$\langle\Omega\rangle = \int \Psi^*\Omega\Psi dx. \tag{21}$$

An analysis of the free particle provides insight into the meaning of $m$. The Stueckelberg equation for the free particle is

$$i\hbar\frac{\partial\psi_f}{\partial s} = -\frac{\hbar^2}{2m}\partial_\mu\partial^\mu\psi_f \tag{22}$$

with the general solution

$$\begin{aligned} i\Psi_f(x,s) &= \int \psi_{f\kappa}(x,s)dk_f \\ &= \int \eta_{f\kappa}\exp\left[i\kappa_f\left(k_f\right)s + ik_{f\mu}x^\mu\right]dk_{f.} \end{aligned} \tag{23}$$

The integral is over energy–momentum, $\eta_{f\kappa}$ denotes normalization coefficient for solution $\psi_{f\kappa}$, $\psi_{f\kappa}(x,s) = \eta_{f\kappa}\exp\left[i\kappa_f\left(k_f\right)s + ik_{f\mu}x^\mu\right]$, and

$$\kappa_f\left(k_f\right) = -\frac{\hbar^2}{2m}k_{f\mu}k_f^\mu. \tag{24}$$

The expectation value of the four-velocity of the free particle is

$$\langle V_f^\mu\rangle = \frac{d\langle x_f^\mu\rangle}{ds} = \frac{\langle p_f^\mu\rangle}{m}. \tag{25}$$

Integrating Equation (25) from $s$ to $s+\delta s$ gives the most probable trajectory of the free particle,

$$\delta\langle x_f^\mu\rangle = \frac{\langle p_f^\mu\rangle}{m}\delta s. \tag{26}$$

The observable world-line of the free particle is given by the inner product

$$\delta\langle x_f^\mu\rangle\delta\langle x_{f\mu}\rangle = \frac{\langle p_f^\mu\rangle\langle p_{f\mu}\rangle}{m^2}(\delta s)^2. \tag{27}$$

Solving for $m^2$ gives:

$$m^2 = \frac{\langle p_f^\mu\rangle\langle p_{f\mu}\rangle}{\delta\left\langle x_f^\mu\right\rangle\delta\langle x_{f\mu}\rangle}(\delta s)^2. \tag{28}$$

As in the classical case, the invariant evolution parameter increases monotonically so that $\delta s > 0$. The terms $\langle p_f^\mu \rangle \langle p_{f\mu} \rangle$, $\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle$ can be either timelike $\langle p_f^\mu \rangle \langle p_{f\mu} \rangle > 0$, $\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle > 0$ or spacelike $\langle p_f^\mu \rangle \langle p_{f\mu} \rangle < 0$, $\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle < 0$ for independent space–time coordinates and energy–momentum components. Consequently, $m^2$ is positive for both timelike and spacelike motion because negative signs associated with spacelike motion cancel. Free tachyons in pRQT have real mass.

## 4. Tachyon Creation and Annihilation in pRQT

Bilaniuk et al. [14] showed that classical special relativity does not prohibit faster-than-light (FTL) motion. Classical special relativity does prohibit subluminal particles from becoming superluminal particles and vice versa when one has the relationship,

$$m = \frac{m_0}{\sqrt{1 - \beta^2}}, \ \beta = \frac{v}{c}, \tag{29}$$

between rest mass, $m_0$, and mass $m$ of a classical particle moving at speed $v$. The mass $m$ approaches infinity as $v \to c$ from below or from above. The question remains whether there a mechanism that makes it possible to avoid the infinite mass prohibition.

Non-relativistic quantum theory suggests an analogous mechanism. In the non-relativistic quantum case, time-dependent interaction potentials enable transitions between energy states, which are observed in such physical systems as spectra and lasers. A study of mass state transitions at a scattering vertex [1,12] showed that the analogous mechanism in pRQT is the transition between mass states associated with $s$-dependent interaction potentials.

It was previously shown [12] that the mass state transition in pRQT can be used as a mechanism for quantum transitions across the light cone. The mechanism enables tachyon creation, annihilation, and detection. An example of a physical system that can be modelled as a mass state transition is the scattering of a particle beam by a thin foil:

$$\text{projectile } (\Psi) + \text{target } (\Phi_T) \rightarrow \text{product.} \tag{30}$$

The simple model of a particle beam-thin foil system, represented by Equation (30), generates mass state transitions between bradyon and tachyon states. The terms "bradyon", "luxon", and "tachyon" denote subluminal, luminal, and superluminal particles, respectively.

A particle beam-thin foil model [12] has the field equation,

$$i\hbar \frac{\partial \Psi}{\partial s} = -\frac{\hbar^2}{2m_\Psi} \frac{\partial^2 \Psi}{\partial x^\mu \partial x_\mu} + g(\Phi_T + \Phi_T^*)\Psi \,, \tag{31}$$

with a postulated coupling constant $g$. The dependence on $s$ occurs in the interaction term $g(\Phi_T + \Phi_T^*)\Psi$. Section 4.1 reviews the perturbation theory for an $s$-dependent perturbing potential in pRQT. The iterative technique for calculating matrix elements for an $s$-dependent perturbation potential is analogous to the non-relativistic iterative solution for a $t$-dependent perturbation. The non-relativistic theory views transition amplitudes for a $t$-dependent perturbation as transitions between energy states, while transition amplitudes for an $s$-dependent perturbation in pRQT refer to transitions between mass states. Section 4.2 applies the formalism to the particle beam–thin foil model.

### 4.1. Perturbation Theory for an s-Dependent Perturbing Potential

The scalar field equation for a perturbation analysis of transitions between mass states due to an $s$-dependent perturbing potential $V_I$ is:

$$i\hbar \frac{\partial \Phi(x_1, s)}{\partial s} = [K_0 + V_I]\Phi(x_1, s) = K\Phi(x_1, s), \tag{32}$$

for spacetime coordinates $x_1$ and invariant evolution parameter $s$. The operator $K_0$ does not depend on $s$. The dependence on $s$ is contained in the Hermitian, perturbing interaction $V_I$. The Hermitian requirement ensures that matrix elements involving $V_I$ are real and meaningful. $s$-dependent perturbation theory in pRQT is analogous to time-dependent perturbation theory used for perturbatively solving the time-dependent Schroedinger equation.

The state $\Phi$ is written as the summation over $N$ mass states as

$$\Phi(x_1, s) = \sum_{n=1}^{N} a_n(s)\phi_n(x_1, s) \tag{33}$$

where $\phi_n(x_1, s),\ n = 1, 2, \ldots, N$, are solutions of the unperturbed field equation,

$$i\hbar \frac{\partial \phi_n(x_1, s)}{\partial s} = K_0 \phi_n(x_1, s). \tag{34}$$

The normalization condition on $\Phi$,

$$\int \Phi^* \Phi d^4 x_1 = 1, \tag{35}$$

Implies:

$$\sum_{n=1}^{N} a_n^*(s) a_n(s) = 1. \tag{36}$$

The set of differential equations for the expansion coefficients is

$$i\hbar \frac{\partial a_m}{\partial s} = \sum_{n=1}^{N} V_{mn} a_n,\ m = 1,\ 2,\ \ldots, N, \tag{37}$$

and the set of matrix elements $\{V_{mn}\}$ of the perturbing potential is

$$V_{mn} = \int \phi_m^* V_I \phi_n d^4 x_1. \tag{38}$$

The field equation for an $s$-dependent perturbation is

$$i\hbar \partial_s \Psi = K_0 \Psi + K_1 \Psi, \tag{39}$$

where $K_0$ refers to the unperturbed mass generator and $K_1$ corresponds to the $s$-dependent interaction term $V_1$ in Equation (32). The perturbation is Hermitian if it satisfies the constraint

$$\int [K_1^* \Psi - \Psi^* K_1] d^4 x = 0. \tag{40}$$

An approximate solution to the perturbation problem is obtained by the eigenfunction expansion,

$$\Psi(x, s) = \int a_\xi(s) \psi_\xi(x, s) d\xi, \tag{41}$$

where $a_\xi(s)$ are expansion coefficients and $\psi_\xi$ are solutions for the unperturbed system.

### *4.2. Particle Beam–Thin Foil Model*

The transition probability amplitude in pRQT is

$$a_\xi = a_\xi^0 - \frac{i}{\hbar} \int_0^s [\int \psi_\xi^* K_1 d^4 x] ds'. \tag{42}$$

The transition probability density to state $\xi$ is

$$P_\xi = a_\xi^* a_\xi \tag{43}$$

and the transition rate density to state $\xi$ is

$$R_\xi = \frac{\partial P_\xi}{\partial s}. \tag{44}$$

The formalism of $s$-dependent perturbation theory applied to the interaction $K_1\Psi = g(\Phi_T + \Phi_T^*)\Psi$ in Equation (31) yields the four-momentum constraints

$$\begin{array}{c} \text{four} - \text{momentum}: \\ k_\alpha = k_a + K_b, \\ k_\alpha = k_a - K_b, \end{array} \tag{45}$$

where subscripts $a, b, \alpha$ denote the projectile, the target, and the product particle, respectively. In addition, there is a set of mass constraint equations:

$$\begin{array}{c} \text{mass}: \\ q_\alpha = q_a + Q_b\,, \\ q_\alpha = q_a - Q_b\,. \end{array} \tag{46}$$

Free particle masses are obtained by solving Equation (39) without the interaction term $K_1$. The result is

$$q_n = \frac{\hbar^2 k_n \cdot k_n}{2m_n\hbar}. \tag{47}$$

The scalar product is

$$k_n \cdot k_n = (k_n)_\sigma (k_n)^\sigma, \tag{48}$$

where the Einstein summation convention applies to index $\sigma$.

Possible mass state transitions are given in Table 1, which is replicated here from references [1,12] for ease of reference. Letter "B" denotes bradyon and letter "T" denotes tachyon. Cases may be signified by the three-letter designation $L_1L_2L_3$, where $L_1$ denotes the projectile particle as either B or T, $L_2$ denotes the target particle as either B or T, and $L_3$ denotes the product particle as either B or T.

The first row in the table is case BBB. In this case, the Product-1 constraint yields a bradyon product after the bradyon projectile interacts with a bradyon target. The product-2 constraint can yield either a bradyon or a tachyon product after a bradyon projectile interacts with a bradyon target. If a bradyon is produced, the case is denoted as BBB. On the other hand, the case is denoted BBT if a tachyon is produced. The product of the interaction depends on the target and projectile properties. Allowed interactions must satisfy a mass constraint and four-momentum constraints. Equation (47) is a mass constraint that constrains physically allowable processes and makes the pRQT formulation different from other tachyon kinematic systems.

**Table 1.** Possible mass state transitions. "B" stands for bradyon and "T" stands for tachyon. The subscripts $a, b, \alpha$ denote the projectile, the target, and the product particle, respectively, the variables $q$ and $Q$ denote the corresponding masses. See text for details.

| **Target**<br>$Q_b$ | **Projectile**<br>$q_a$ | **Product-1**<br>$q_\alpha = q_a + Q_b$ | **Product-2**<br>$q_\alpha = q_a - Q_b$ |
|---|---|---|---|
| B | B | B | B<br>T |
| B | T | B<br>T | T |
| T | B | B<br>T | B |
| T | T | T | B<br>T |

## 5. Projectile-Stationary Target Kinematics

The allowed physical properties of tachyons in the simple particle beam–thin foil model considered are constrained by kinematic equations. Consequently, an experimental search must be very specific. As an example, consider particle experiments that involve the interaction of a projectile beam with a thin foil. Except for thermal motion, it is reasonable to assume that target particles in the foil may be viewed as stationary targets relative to an energetic incident beam. The example presented here shows that tachyon creation and annihilation are kinematically possible in pRQT using the equations and detailed calculations given in [1,12]. The calculations have been updated using particle properties from [13].

The equations for tachyon creation following the interaction of bradyons (case BBT) are:

$$\begin{aligned} \alpha_0 &= a_0 - b_0 \,, \\ \alpha_1 &= a_1 - b_1 \,, \\ m_\alpha &= m_b - m_a, \end{aligned} \tag{49}$$

where $a_0, a_1,\ b_0, b_1$ and $\alpha_0, \alpha_1$ are the nonzero components of the energy-momentum four-vectors for the projectile, target, and product particles, respectively. The masses of the projectile, target, and product particles are $m_a, m_b,$ and $m_\alpha$, respectively. The mass constraint is simplified by noting that observable free particles have sharply peaked expectation values (see [12], Section 4.2). In addition to Equation (49), the on-shell mass-energy-momentum relations,

$$\begin{aligned} \text{bradyon projectile}: \ m_a^2 &= a_0^2 - a_1^2, \\ \text{bradyon target}: \ m_b^2 &= b_0^2 - b_1^2, \\ \text{tachyon product}: \ m_\alpha^2 &= \alpha_1^2 - \alpha_0^2 \end{aligned} \tag{50}$$

must be satisfied.

Results for the scattering of a pion projectile by a proton target are shown in Table 2. The applicability of the scalar particle model is improved here by using the electrically neutral pion as the incident projectile so there is no electromagnetic interaction. The resulting model provides kinematic relations for the interaction of a neutral pion projectile with a proton target via a hypothesized $s$-dependent potential. The product particle is an anti-tachyon with real mass. Details of the case BBT calculation are outlined in Appendix A.

**Table 2.** Case BBT calculation (in MeV/$c^2$ with $c$ set to 1) with Equations (49) and (50) satisfied up to four digits after the decimal shown. See text for details.

| Process | Projectile | Target | Product |
|---|---|---|---|
| $\pi^0 + p$ | $a_0 = 822.7126$<br>$a_1 = 811.5647$<br>$m_a = 134.9768$ | $b_0 = 938.2721$<br>$b_1 = 0.0000$<br>$m_b = 938.2721$ | $\alpha_0 = -115.5595$<br>$\alpha_1 = 811.5647$<br>$m_\alpha = 803.2953$ |

Table 2 shows that experimentally attainable energies and readily available bradyons are theoretically suitable for producing tachyons (or anti-tachyons). If our model is physically realistic, it is possible that tachyons may have already been created but not detected. The results presented in Table 2 require that a detector must be tuned to either detect a case BBT superluminal anti-particle or a case TBB bradyon particle. An example of a set of properties that might be expected for a case TBB bradyon can be obtained by using the tachyon produced in the neutral pion–proton interaction shown in Table 2.

The case TBB equations for tachyon annihilation following the interaction with a bradyon are:

$$\begin{aligned} \alpha_0 &= a_0 + b_0, \\ \alpha_1 &= a_1 + b_1, \\ m_\alpha &= m_b - m_a \,, \end{aligned} \tag{51}$$

where $\{a_0, a_1\}$, $\{b_0, b_1\}$, and $\{\alpha_0, \alpha_1\}$ are the nonzero components of the energy–momentum four-vectors for the projectile, target, and product particles, respectively. In addition to Equation (51), the on-shell mass-energy-momentum relations are:

$$\begin{aligned} \text{tachyon projectile}: \; m_a^2 &= a_1^2 - a_0^2 \,, \\ \text{bradyon target}: \; m_b^2 &= b_0^2 - b_1^2 \,, \\ \text{bradyon product}: \; m_\alpha^2 &= \alpha_0^2 - \alpha_1^2 \,, \end{aligned} \tag{52}$$

where masses $m_a, m_b$, and $m_\alpha$ must be satisfied. Assuming the case BBT product anti-tachyon is the case TBB anti-tachyon projectile, and taking the case TBB target particle to be a proton, the two solutions are found as shown in Tables 3 and 4. Solution 1 refers to a stationary target, and Solution 2 refers to a non-stationary target. Details of the case TBB calculation are outlined in Appendix B.

**Table 3.** Case TBB Solution 1 (in MeV$/c^2$ with $c$ set to 1) with Equations (51) and (52) satisfied up to four digits after the decimal shown. See text for details.

| Process | Projectile | Target | Product |
|---|---|---|---|
| $T + p$ | $a_0 = -115.5595$<br>$a_1 = 811.5647$<br>$m_a = 803.2953$ | $b_0 = 938.2721$<br>$b_1 = 0.0000$<br>$m_b = 938.2721$ | $\alpha_0 = 822.7126$<br>$\alpha_1 = 811.5647$<br>$m_\alpha = 134.9768$ |

**Table 4.** Case TBB Solution 2 (in MeV$/c^2$ with $c$ set to 1) with Equations (51) and (52) satisfied up to four digits after the decimal shown. See text for details.

| Process | Projectile | Target | Product |
|---|---|---|---|
| $T + p$ | $a_0 = -115.5595$<br>$a_1 = 811.5647$<br>$m_a = 803.2953$ | $b_0 = -977.1067$<br>$b_1 = 272.7325$<br>$m_b = 938.2721$ | $\alpha_0 = -1092.6662$<br>$\alpha_1 = 1084.2971$<br>$m_\alpha = 134.9768$ |

## 6. Discussion

We have shown that formulations of relativistic classical and quantum mechanics with an invariant evolution parameter predict that tachyon mass should be real. Illustrative calculations of creation, annihilation, and detection of tachyons using a beam of particles interacting with a thin foil are presented using up-to-date data. The cases, presented in Section 5, show that scattering of a particle beam by a thin foil provides a means for designing an experiment capable of producing (case BBT) and detecting (case TBB) tachyons.

Tachyon physics in pRQT [1,4,9,12] differs from other formulations. Many alternative theories are available in the literature. Introductions to alternative theories are presented in Refs. [15–21]. More recent examples of alternative theories are presented in Refs. [22–34]. They cover a range of topics, including expansion of the universe [16], inflation [22–24,31,33], the cosmological constant [25,26], dark energy [29], spontaneous symmetry breaking [19,27], rolling tachyons [15,17], trapped ions [30], condensation in magnetic compactification [34], and the possibility of tachyonic neutrinos [28,32].

Pavšič [4] clarified and analyzed tachyon topics that he identified as misconceptions and confusing. For example, the conventional view is that tachyons have imaginary mass. By contrast, tachyons in pRQT have real mass. Furthermore, mass-state transitions in pRQT provide a mechanism for creating, annihilating, or detecting tachyons. Comparisons of pRQT with alternative theories suggest directions for future work in pRQT. They also provide an opportunity for comparing and testing different theories, as illustrated in [35,36].

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## Appendix A. Case BBT Calculation Procedure

The calculation procedure, provided in a previous paper [12], is replaced by a more direct solution procedure. Case BBT constraint equations are Equations (49) and (50). In this case, the following variables are known:

$$\text{masses } m_a, m_b, \text{ and } b_1 = 0. \tag{A1}$$

From Equations (49) and (50),

$$\begin{aligned} m_\alpha &= m_b - m_a, \\ b_0 &= m_b. \end{aligned} \tag{A2}$$

From Equations (49) and (A2),

$$\begin{aligned} a_0 - \alpha_0 &= b_0, \\ \alpha_0 &= a_0 - m_b, \\ \alpha_1 &= a_1 \text{ since } b_1 = 0. \end{aligned} \tag{A3}$$

From Equations (50) and (A2),

$$m_\alpha^2 = (m_b - m_a)^2 = \alpha_1^2 - \alpha_0^2. \tag{A4}$$

Insert Equation (A3) into the RHS of Equation (A4):

$$(m_b - m_a)^2 = \alpha_1^2 - (a_0 - m_b)^2. \tag{A5}$$

Solve for $a_1$ using Equations (50) and (A5):

$$a_1^2 = (m_b - m_a)^2 + (a_0 - m_b)^2 = a_0^2 - m_a^2. \tag{A6}$$

Solve for $a_0$ by expanding Equation (A6):

$$a_0 = \frac{1}{2m_b}\left[m_a^2 + (m_b - m_a)^2 + m_b^2\right]. \tag{A7}$$

The remaining terms are found from

$$\begin{aligned} a_1^2 &= a_0^2 - m_a^2, \\ \alpha_0 &= a_0 - m_b, \\ \alpha_1 &= a_1. \end{aligned} \tag{A8}$$

Results are verified by showing that Equations (49) and (50) are satisfied.

## Appendix B. Case TBB Calculation Procedure

Case TBB constraint equations are Equations (51) and (52). In this case, the following variables are known:

$$\text{masses } m_a, m_b \text{ and tachyon values } a_0 \textit{ and } a_1. \tag{A9}$$

Tachyon values are given by case BBT. Confirm

$$m_a^2 = a_1^2 - a_0^2 \tag{A10}$$

and calculate

$$m_\alpha = m_b - m_a. \tag{A11}$$

From Equations (51) and (52), find

$$m_\alpha^2 = \alpha_0^2 - \alpha_1^2 = (a_0 + b_0)^2 - (a_1 + b_1)^2 \tag{A12}$$

and

$$m_b^2 = b_0^2 - b_1^2 \Longrightarrow b_0^2 = m_b^2 + b_1^2. \tag{A13}$$

Rearrange Equation (52) to get

$$b_0^2 = m_b^2 + b_1^2 \Longrightarrow b_0 = \pm\sqrt{(m_b^2 + b_1^2)}. \tag{A14}$$

Since $b_0, b_1$ are unknown but constrained by Equation (A13), assume

$$b_1 = \lambda m_b \tag{A15}$$

for parameter $\lambda$. Substitute Equation (A15) into Equation (A13) so that

$$b_0^2 - b_1^2 = b_0^2 - (\lambda m_b)^2 = m_b^2. \tag{A16}$$

Rearrange Equation (A16) for

$$b_0^2 = m_b^2\left(1 + \lambda^2\right). \tag{A17}$$

Combine Equations (A11) and (A12) to get

$$m_\alpha^2 = (m_b - m_a)^2 = (a_0 + b_0)^2 - (a_1 + b_1)^2. \tag{A18}$$

Substitute the solution $b_0 = -m_b\sqrt{(1 + \lambda^2)}$ of Equation (A17), based on Ref. [12], into Equation (A18) to get

$$(m_b - m_a)^2 = \left(a_0 - m_b\sqrt{\left(1 + \lambda^2\right)}\right)^2 - (a_1 + \lambda m_b)^2. \tag{A19}$$

Expand Equation (A19) and simplify to get

$$a_0 m_b\sqrt{\left(1 + \lambda^2\right)} = m_b m_a - m_a^2 - a_1\lambda m_b. \tag{A20}$$

The term $\lambda$ is found as the intersection of the left-hand side and right-hand side of Equation (A20) as functions of $\lambda$. This value of $\lambda$ is used in Equations (A15) and (A17) and $b_0 = -m_b\sqrt{(1 + \lambda^2)}$ to get $\{b_0, b_1\}$. The terms $\{\alpha_0, \alpha_1\}$ are found from Equation (52) to complete case TBB.

Results are verified by showing that Equations (51) and (52) are satisfied.

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