education sciences

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S^{3}D Approach: Incremental Professional Development for Fostering Small-Group Discourse

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Abstract: The value of discourse in the mathematics classroom is evident across standards, policy documents, and research. The quality of discourse is associated with students' mathematical understanding and achievement. Productive mathematical discourse includes students providing explanations and critically listening to and evaluating the ideas of others. Such high-level discourse takes time to develop and is impacted by how teachers structure lessons and interact with students (e.g., talk moves). The present case study reports on an incremental professional development, S^{3}D Approach, that guides teachers through a two-phase process to enhance small-group, student-to-student discourse. One middle school mathematics teacher implemented the incremental stages of the S^{3}D Approach with support from a mathematics teacher educator, who conducted weekly observations and debrief meetings. Qualitative analyses revealed that, despite initial challenges, the S^{3}D Approach became integrated into the teacher's practice, enabling the teacher to identify and build upon incremental improvements in the small-group discourse. Overall, the findings demonstrate how through a reframing of starting points and the definition of success, incremental PD supports sustainable changes in a teacher's practice and student engagement in productive mathematical discourse.

Keywords: mathematical discourse; small groups; talk moves; incremental professional development

Academic Editors: Federico Corni, Samuel Otten, Amber Candela and Zandra De Araujo

Received: 15 September 2024
Revised: 7 December 2024
Accepted: 24 December 2024
Published: 31 December 2024

Citation: Quebec Fuentes, S. (2025). S^{3}D Approach: Incremental Professional Development for Fostering Small-Group Discourse. *Education Sciences*, *15*(1), 36. https://doi.org/10.3390/educsci15010036

1. Introduction

To be responsible citizens with the ability to engage in lifelong learning, students need to develop more than just content knowledge (DeSeCo, 2024). Under this premise, the OECD initiated the Definition and Selection of Competencies [DeSeCo] (2016) Program to create a framework of key competencies (Organisation for Economic Co-operation and Development [OECD], 2005). The framework consists of three categories, one of which is to interact in heterogenous groups. Similarly, Ananiadou and Claro (2009) developed a three-dimensional framework for 21st century skills with one of the dimensions being communication. "Communication plays an important role in the preparation of students to be not only lifelong learners, but also members of a larger community with voice and a sense of responsibility to others. Young people need to have the ability to communicate, exchange, criticize, and present information and ideas" (Ananiadou & Claro, 2009, p. 10).

In the context of the learning and teaching of mathematics, opportunities to collaborate and interact in heterogenous groups contribute to the development of students' reasoning capabilities and conceptual understanding of mathematics (National Council of Teachers of Mathematics [NCTM], 2014; National Research Council [NRC], 2012; Organisation for Economic Co-operation and Development [OECD], 2021, 2023). Bishop (2021) claims, "Learning occurs in interaction. And one of the primary means of interaction in classrooms

is discourse" (p. 467). The value of discourse in the mathematics classroom is evident across standards, policy documents, and research. The nature and quality of discourse impacts "what students learn about mathematics as well as how they learn it" (National Council of Teachers of Mathematics [NCTM], 1991, p. 54). In the United States, communication is one of the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2000) process standards and is reflected in one of the Common Core State Standards for Mathematical Practice (SMP), Construct viable arguments and critique the reasoning of others (SMP 3) (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010). The standards and curricula of other countries also address the importance of mathematical discourse (Ing et al., 2015) (e.g., Standards for Excellence in Teaching Mathematics in Australian Schools (Australian Association of Mathematics Teachers [AAMT], 2006)).

Helping students develop their ability to interact with others enhances their level of participation (Mercer & Sams, 2006). In turn, the level of student participation is positively associated with the development of student understanding and achievement (Warner, 2008; Webb et al., 2014). For example, when teachers restated the ideas of students, encouraged students to share explanations, and questioned student reasoning, the students provided more detailed explanations and scored higher on assessments of their reasoning (Gillies & Haynes, 2011). In fact, discourse is a mediating factor between teachers fostering such discourse and student achievement (Bishop, 2021; Ing et al., 2015).

Discourse is "the spoken and written words, representations, and gestures people use to communicate, interact, and act" (Bishop, 2021, p. 468). The primary focus of the incremental professional development (PD) shared herein is on developing student-to-student talk in small groups. "Students must talk with one another . . . ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings within an intellectual community" (National Council of Teachers of Mathematics [NCTM], 1991, p. 34). Student actions that involve high levels of intellectual work (Bishop, 2021) include making conjectures, reasoning logically, and constructing viable arguments (Bishop, 2021; National Council of Teachers of Mathematics [NCTM], 1991; National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010); knowing if others understand their arguments (National Council of Teachers of Mathematics [NCTM], 2000); listening to, attempting to understand, and evaluating the arguments of others (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010; Ing et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); trying and building upon the strategies of others (National Council of Teachers of Mathematics [NCTM], 2000, 2014); and comparing strategies and making connections between ideas and various representations (Bishop, 2021; Franke et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014).

Further, "discourse should be focused on making sense of mathematical ideas" (National Council of Teachers of Mathematics [NCTM], 1991, p. 45). Therefore, students must explain their ideas and reasoning (Bishop, 2021; National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010; Franke et al., 2015; Ing et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); ask clarifying questions (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010; Ing et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); provide critiques of others' correct and incorrect thinking with justifications (Bishop, 2021; National Council of Teachers of Mathematics [NCTM], 1991, 2014); reply to these critiques (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010); describe the strategies of others and propose strategies (National Council

of Teachers of Mathematics [NCTM], 1991, 2000, 2014); and ask classmates to engage in the aforementioned actions (Bishop, 2021). For productive or meaningful mathematical discourse to occur (National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014), the conversation builds on what has been said and done (Barron, 2000).

A communication-rich mathematics classroom (National Council of Teachers of Mathematics [NCTM], 2000) with students engaged in the aforementioned actions and interactions develops over time (National Council of Teachers of Mathematics [NCTM], 1991). Hufferd-Ackles et al. (2004) describe levels through which teachers and students progress as they develop a discourse community. Students transition from providing closed answers to teacher questions to sharing their reasoning. Students progress to engaging in student-to-student talk with teacher support and then independent of the teacher (Hufford-Ackles et al., 2014). Webb et al. (2014) differentiate between low, medium, and high levels of student engagement with others' ideas. Low-level engagement occurs when a student's reference to another's idea does not include any mathematical details, and medium-level engagement involves the reiteration of another student's idea without elaboration. In contrast, when students engage at a high level, they build upon correct conceptions, address misconceptions, or collaborate. The level of engagement with others' reasoning is positively correlated with achievement.

Teachers help students move toward these higher levels of engagement. One of the eight effective teaching practices in NCTM's *Principles to Actions* is to facilitate meaningful mathematical discourse: "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (National Council of Teachers of Mathematics [NCTM], 2014, p. 29). The teacher orchestrates discourse (National Council of Teachers of Mathematics [NCTM], 1991, p. 34) in the mathematics classroom using a two-pronged approach (Steele, 2019). First, teachers must set goals, choose tasks, and structure lessons to provide a foundation for high levels of engagement (e.g., Smith & Stein, 2018). Second, teachers need to interact with students when they engage with tasks and share their approaches with others.

Researchers name or categorize various teacher interactions in different ways. B. A. Herbel-Eisenmann et al. (2013) describe six Teacher Discourse Moves: waiting, inviting, revoicing, asking students to revoice, probing a student's thinking, and creating (opportunities to engage with another's reasoning) (p. 183). Candela et al. (2020) differentiate between two categories of Discourse Actions. Teacher press actions prompt students to explain and justify their reasoning, and teacher linking actions encourage students to attend to others' thinking. Franke et al. (2015) distinguish between invitation moves that initiate engagement with each other's ideas and support moves that enhance the engagement. To encourage student-to-student talk in small groups, Quebec Fuentes (2020) identifies specific talk moves to address various group dynamics.

Chapin et al. (2013) define talk moves as, "strategic ways of asking questions and inviting participation in classroom conversations" (p. 11). A common feature across all categorizations of teacher interactions, or talk moves, is the goal for students to critically listen and respond to the thinking of peers. To meet this goal, Chapin et al. articulate four steps to classify the intention of talk moves:

- Step 1: Helping Individual Students Clarify and Share Their Own Thoughts (e.g., wait time, turn and talk, say more, revoicing)
- Step 2: Helping Students Orient to the Thinking of Others (e.g., Who can put that into their own words?)
- Step 3: Helping Students Deepen Their Own Reasoning (e.g., Why do you think that?)

- Step 4: Helping Students Engage with the Reasoning of Others (e.g., Do you agree or disagree . . . and why? Who can add on?) (p. 10)

Bishop (2021) found a significant and positive relationship between teacher interactions that helped students to engage with classmates' ideas and student learning. However, such interactions did not happen often.

"Engaging every student in the discourse of the class requires considerable skill" (National Council of Teachers of Mathematics [NCTM], 1991, p. 34). Teachers must establish both general norms around the role of the teacher and students and a culture of collaboration (Manouchehri & Enderson, 1999) and sociomathematical norms (Yackel & Cobb, 1996). Stein (2007) uses the term "motivational discourse" to build norms around student involvement in discourse, such as viewing mistakes as learning opportunities and engaging in productive struggle (National Council of Teachers of Mathematics [NCTM], 2014). To transition to such norms, teachers must alter the classroom talk from being focused on teacher thinking (e.g., univocal or funneling) to being centered on the reasoning of students (e.g., dialogic or focusing) (Knuth & Peressini, 2001; Wood, 1998). Small-group discourse presents additional challenges, as a teacher cannot be present with all groups at all times (Quebec Fuentes, 2020). PD can support teachers in this large-scale undertaking. Examples of such ambitious PD includes the yearlong experience of high school teachers who learned about and then conducted action research centered on classroom discourse (B. Herbel-Eisenmann & Cirillo, 2009) and the two-year Mathematics Discourse in Secondary Classrooms PD that involves 40 to 50 h for participation in a study group followed by a capstone project (B. A. Herbel-Eisenmann et al., 2013, 2017).

As an alternative to ambitious PD, Otten et al. (2022) propose incremental PD, which involves smaller changes to familiar teacher practices. Quebec Fuentes (2013), as a secondary mathematics teacher, conducted action research to study her own practice with respect to improving small-group discourse. Through repeated action research cycles, she discovered that she could not haphazardly utilize talk moves. Instead, she first needed to assess the dynamics of a group, the nature and quality of student-to-student talk within the group, and her instinctual ways of interacting with the group. With this information, she was able to purposefully choose talk moves that addressed a particular group's dynamics to effectively enhance the student-to-student discourse. Based on these findings, Quebec Fuentes (2020, 2022) developed and implemented three iterations of an incremental PD, called the S^3D Approach, to support teachers who want to foster Small-group, Student-to-Student Discourse. Although the overarching goal of developing productive small-group discourse may be considered ambitious, the S^3D Approach is broken down so that teachers attend to various aspects of small-group discourse individually and over time. In particular, the S^3D Approach consists of two phases. In Phase 1, the teacher assesses the current discourse of small groups through three lenses (Table 1).

Teachers use what they learn from the three lenses of Phase 1 as they move on to Phase 2. They set goals for groups, plan Process Help talk moves that align with each group's dynamics, implement the talk moves, and reflect on the impact of the talk moves. Teachers are encouraged to maintain the same groups for a period of time (e.g., 10 weeks) and start small by initially implementing the S^3D Approach with one to two focus groups. When teachers change groups, they repeat both phases with the new groups. Additionally, tools support teachers' assessment at each phase/lens. Refer to Appendix A for each of the tools that contain detailed information about each phase/lens.

The purpose of the present study is to examine the implementation of the S^3D Approach from the perspective of incremental PD. The study focuses on the most recent implementation of the S^3D Approach with one teacher; therefore, a qualitative case study

design was used (Merriam & Tisdell, 2016). Specifically, the following research questions were addressed:

1. What is the teacher's experience when engaging in incremental PD?
2. How do the tools support the teacher as she progresses through the incremental PD?
3. How does the teacher's practice change over time?

Table 1. Descriptions of the three lenses involved in Phase 1of the S^3D Approach (Quebec Fuentes, 2020, 2022).

Lens	Description
Group Dynamics	The Group Dynamics are ways in which students behave in small groups that inhibit productive mathematical discourse. The 10 dynamics (e.g., Non-Participatory Student, Dominant Student, Rush to Complete Task) reflect ways that groups initially interact and provide a starting point for transitioning toward more productive discourse.
Discourse Quality	The Discourse Quality is the nature and quality of the student-to-student talk. Students' questions and their peers' responses are categorized according to eight types of question–response (QR) pairs that increase in their level of cognitive demand (e.g., questions focused on logistics, answers, explanations, or critiques).
Teacher Support	Teacher Support refers to how a teacher interacts with a group, differentiating between Product Help and Process Help (Dekker & Elshout-Mohr, 2004). Product Help focuses on the mathematics content and often results in a conversation between the teacher and only one student (e.g., funneling). Process Help focuses on helping the students communicate with each other about the mathematics.

2. Materials and Methods

2.1. Participants and Setting

The present article reports on the third iteration of the PD, which was conducted with one middle school mathematics teacher, Arden (pseudonym), the primary participant of the study. Arden had been a student of the author in her graduate studies and volunteered to work on fostering small-group discourse with the author, a mathematics teacher educator (MTE) who both conducted the professional development and the research surrounding its implementation. Arden was in her second year of teaching after completing an undergraduate degree in middle school mathematics education and a master's degree in mathematics education. She taught in a classical online charter school in the Southwest United States. Classes at the school are taught synchronously via Zoom. The class periods were 45 min in length. Arden's lessons typically involved a whole-class lesson launch, small-group work on a task, and then a whole-class summary of connections between the groups' solution strategies and mathematical concepts underlying the tasks (Smith & Stein, 2018). Arden used Google Jamboard as a platform to facilitate all three phases of the lesson. Each group had a shared Jamboard slide; one group member shared their screen in the Zoom breakout room. During small-group work, Arden could see each group's work on Jamboard and visited the various groups as they collaborated in the breakout rooms.

As part of the professional development, Arden chose to center her work on fostering small-group discourse in her two seventh grade mathematics classes. Each class had 15 to 25 students (11 to 12 years of age), who were divided into groups consisting of 2 to 4 students. In the first quarter at the beginning of the year, the students were randomly assigned to groups. In the subsequent quarters, Arden purposefully grouped students based on what she had learned about how they interacted in groups using a variety of reasons (e.g., grouping a shy student with more collaborative and supportive peers). Students were physically located in the same state, but classes took place at their respective homes. Since the school was in its second year of operation, many of the students came from other charter and public in-person schools, and several were previously homeschooled. The student body comprised individuals from various ethnic backgrounds, including

Asian, African American, Hispanic, and White. In Arden's seventh-grade pre-algebra class, much of the year was spent delving into proportionality. Specifically, the class addressed scale factors as percentages, coordinate rules for the enlargement and reduction of shapes, proportional relationships to make predictions, graphs and tables, and equations. Additionally, students learned preliminary data analysis, constructing and measuring, and function foundations. The school adopted Carnegie curriculum materials, and the teacher supplemented it with Connected Mathematics Project Grade 7 throughout the year.

2.2. Procedure

Arden participated in the PD each quarter, for which she created new groups that remained together for the 10 weeks. She chose to focus on two groups, one from each seventh-grade class, during each quarter. To identify the focus groups, Arden used the Group Dynamics tool to identify groups manifesting multiple dynamics inhibiting productive discourse. The PD was cyclical and broken into incremental phases according to the S^{3}D Approach (Figure 1). In Phase 1, Arden assessed the current discourse of the focus groups through the three lenses: Group Dynamics, Discourse Quality, and Teacher Support. For each lens (left side of Figure 1), Arden learned about that component of small-group discourse and the associated tool. She used the tool during and after her lessons to document her observations and then participated in a weekly debrief session during which she shared what she learned about the discourse of the focus groups with respect to that particular lens. She repeated this cycle for the two other lenses in Phase 1. Arden then progressed to Phase 2 (right side of Figure 1). She used all of her reflections from the three lenses of Phase 1 to set goals for each focus group and plan the talk moves she would use, implemented the talk moves in practice, reflected on the impact of the talk moves on the student-to-student discourse, and shared her reflections in the weekly debrief session. This Phase 2 cycle was repeated for the remainder of the quarter. Arden repeated the entire process each quarter, restarting with the assessment of the Group Dynamics of the new groups (i.e., first lens of Phase 1).

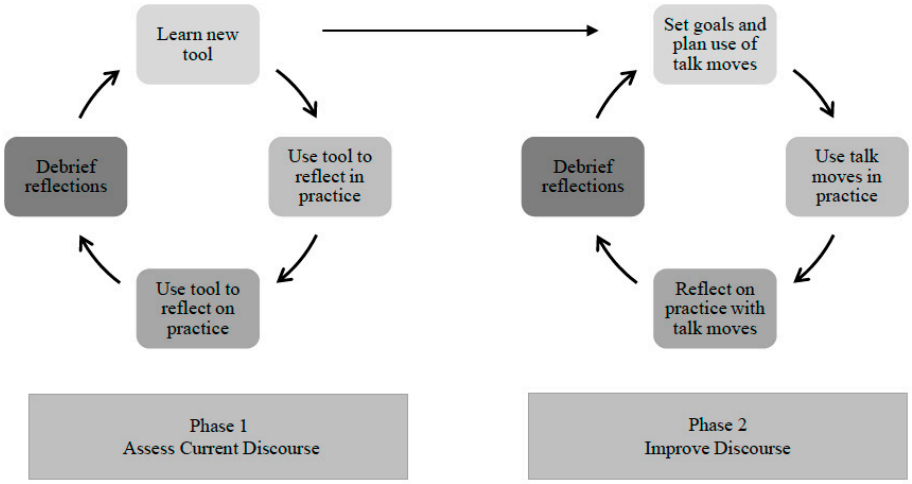

Figure 1. Cyclical implementation of the PD (Quebec Fuentes, 2020).

The MTE observed each seventh-grade class one time per week. She observed the entirety of the lesson (whole-group launch, small-group work, and whole-group summary). During the small-group work, the MTE observed the focus group in their breakout room. The MTE took fieldnotes (i.e., a script of the lesson interactions) during the observations and documented the nature of the small-group discourse using the four tools associated with the S^{3}D Approach (Group Dynamics, Discourse Quality, Teacher Support, and Talk Moves) after the observation. At the end of each week, the MTE debriefed with Arden. The debrief

meetings had several purposes. During each conversation, Arden shared her reflections on the small-group discourse with respect to the current phase/lens and her experience engaging with the S^{3}D Approach, and the MTE provided feedback as well. At the close of each session, Arden and the MTE decided whether to continue with the current cycle or transition to the next cycle. In the latter case, the MTE introduced Arden to the new phase or lens and the associated tool. The MTE video-recorded each debrief session and took fieldnotes.

2.3. Data Analysis

The data sources included the observation fieldnotes and the videos and fieldnotes from the debrief meetings. The MTE used the observation fieldnotes to analyze the four aspects of the small-group, student-to-student discourse. First, the MTE coded the observation fieldnotes with respect to Group Dynamics (10 group dynamics), Discourse Quality (eight question–response pairs), Teacher Support (Product Help and Process Help), and Talk Moves (type of Process Help talk move). For each observation, the MTE then completed the Group Dynamics, Discourse Quality, Teacher Support, and Talk Moves tools. For each group dynamic coded, the MTE provided a description of how the dynamic manifested. For the Discourse Quality, the MTE tallied each question–response pair and wrote a summary of the nature of the student-to-student talk. For Teacher Support, the MTE indicated whether Arden used Product Help and/or Process Help. The MTE also documented the specific Process Help talk moves used and the impact of those talk moves using the Talk Moves tool. After each 10-week quarter was completed, the MTE synthesized the information documented in the four tools from all observations across the quarter to create a profile of each group, showing the evolution of the student-to-student talk as related to the impact of the teacher Process Help talk moves.

The transcribed videos and fieldnotes from the debrief meetings provided evidence of the teacher's experience engaging with the S^{3}D Approach. The MTE analyzed the fieldnotes using the constant comparative method (Glaser & Strauss, 1967). First, the MTE read through the fieldnotes and openly coded them using the comments feature in Word. The codes were then refined and categorized into five categories: teacher practice as problem solving (e.g., formal reflection, unanticipated outcomes), nature and implications of incremental PD (e.g., choose pace, change in norms), use of tools to enhance teacher practice (e.g., reveal intricacies of small-group discourse, modification of tools), teacher challenges (e.g., managing multiple groups, impatience), and incremental change over time (e.g., not recognizing and recognizing improvement).

3. Results

The present study reports on Arden's experiences during the first two quarters of the academic year. The results are organized according to the five themes (teacher practice as problem solving, nature and implications of incremental PD, use of tools to enhance teacher practice, and teacher challenges and incremental change over time). Codes within each theme are in italics.

3.1. Teacher Practice as Problem Solving

The S^{3}D Approach centers on a problem of practice. In particular, the approach involves understanding the current state of small-group, student-to-student talk and moving toward enhancing the quality of that discourse. The MTE explained the foundations of the process to Arden: "You are actively taking this [problem of practice] on, you are purposefully thinking about it, collecting data, and making decisions off of that data. That is how you are going to see improvement. It will not be tomorrow, but you will have it" (Debrief Meeting 5). The tools associated with the S^{3}D Approach support data collection

and a transition from informal to more formal reflection. Arden explained, "It is interesting as we move through the tools . . ., [the informal] observations are vague until I can look at the tool and see these very specific [aspects of the discourse]" (Debrief Meeting 3). The formal reflection sometimes revealed unexpected information. For instance, when Arden changed groups for the second quarter, she had anticipated that one of the students in a focus group would be non-participatory. Arden expressed her disbelief:

> I noticed . . . when I went into their groups a couple times to observe that Milton was speaking more than I'd ever seen him speak in a group. . . . Lillie is bringing him out a little bit and that was a completely unexpected dynamic. In the beginning you [said] you can predict these dynamics, but it might surprise you. And I'm surprised at how much he's talking right now. (Debrief Meeting 11)

With such targeted reflection, Arden was able to see intricacies of the small-group discourse including *unanticipated outcomes*.

3.2. Nature and Implications of Incremental PD

One of the goals of the S^{3}D Approach is for small groups "to have sustainable conversations when [the teacher] is not there" (MTE, Debrief Meeting 5). Arden expressed the enormity of this goal, "it is a lot of work. . . . I hope it pays off" (Debrief Meeting 8). In response, the MTE explained that there are various factors that influence the nature of small-group talk, "There are so many intricacies and that probably feels overwhelming from the teacher perspective: 'I have so much going on and now I have to be thinking about the intricacies of how they interact.'" (Debrief Meeting 8). To address these *intricacies* and make the process more manageable, the S^{3}D Approach is broken down into two incremental phases, as explained previously. Arden described the approach: "It is completely a science to get them to learn how to talk to each other. And then not just talk, . . . really speak mathematically and then speak meaningfully" (Debrief Meeting 9).

To further make the approach accessible, Arden was able to *choose the pace* at which she progressed through the phases and lenses each quarter. For example, the following conversation about whether to move on to the next Phase 1 lens occurred during Debrief Meeting 5:

> MTE Where do you want to go as you delve into next week?
> Arden Well, I still think I need to live in the Discourse Quality. I need more time in that . . . with juggling the other groups . . . especially since I only gathered two days [of data] this week.

Similar conversations about goals and talk moves occurred during Phase 2.

Arden progressed through the two phases each quarter. In the first quarter, *getting acquainted with the phases, lenses, and associated tools* took time. For instance, Arden shared her experience with implementing talk moves for the first time:

> I'm not taking as many notes as I thought I would. . . . I would reference the talk moves . . . to refresh myself before going to the group. I would find myself doing those moves and then realizing I'm accomplishing a lot by doing this move. . . . [But] I forget to track it; I'm just so in the weeds right now. (Debrief Meeting 9)

By progressing through the two phases during the first quarter, Arden became more familiar with the intricacies of the approach. During Debrief Meeting 13, she described her familiarity with the talk moves: "I still think about them in the back of my head." As a result, she was able to move through the phases more quickly with new groups during the second quarter. Table 2 shows how much time she spent on each aspect of the S^{3}D Approach for the first and second quarters. In the first quarter, Arden started the S^{3}D Approach during the third week. She spent most of the quarter in Phase 1, implementing

the talk moves of Phase 2 for the last two weeks. In the second quarter, Arden quickly progressed to Phase 2 after spending only three weeks in Phase 1.

Table 2. Time spent on each phase/lens in Quarters 1 and 2.

	Phase/Lens	
Week	**Quarter 1**	**Quarter 2**
1		Phase 1—Group Dynamics
2		Phase 1—Group Dynamics
3	Phase 1—Group Dynamics Choose Quarter 1 Focus Groups	Phase 1—Discourse Quality
4	Phase 1—Group Dynamics	Phase 2—Talk Moves
5	Phase 1—Group Dynamics Phase 1—Discourse Quality	Phase 2—Talk Moves
6	Phase 1—Discourse Quality	Phase 2—Talk Moves
7	Phase 1—Teacher Support	Phase 2—Talk Moves
8	Phase 1—Teacher Support	Phase 2—Talk Moves
9	Phase 2—Talk Moves	Phase 2—Talk Moves Choose Quarter 3 Focus Groups
10	Phase 2—Talk Moves Choose Quarter 2 Focus Groups	

The MTE discussed this shift when transitioning to the second cycle: "You could jump [to Phase 2]. I know you're already [implementing talk moves] because I have it in my notes and my data. . . . You're hitting this point where . . . you can go through Phase 1 much faster" (Debrief Meeting 13). The nature of the student-to-student talk also improved more quickly in the second quarter. The MTE shared her observations during the first couple of weeks of the quarter: "There was definitely a lot of student talk going on; whereas, when I started observing at the beginning of the year there wasn't" (Debrief Meeting 9) and "They're still fairly new groups. So, you're going to get a lot of question-response pair 1 [QR1; lower cognitive demand]. . . . [However] we're only two weeks in, and . . . they're [transitioning to QR pairs of higher cognitive demand] pretty fast" (Debrief Meeting 12). Even though the students were in new groups, the benefits of Arden going through the S^3D Approach in Quarter 1 were evidenced in Quarter 2.

Another way the process was incremental is that Arden *chose one group in each class as her focus group*. She implemented the S^3D Approach with the focus groups. When Arden was trying to choose between two groups for one of the Quarter 1 focus groups, the MTE explained:

> This is not an absolute decision. You are going to be working with both of these groups on the regular. Just one of them you are going to be taking that step to collect some data. Eventually everything you will be learning with your focus groups will be integrated into other groups as well.(Debrief Meeting 3)

Working with the focus groups allowed Arden to become acquainted with the process and then consider ways to *extend what she learned to other groups*. Arden and the MTE discussed the act of "juggling" the different groups during Debrief Meeting 5:

> Arden What I want to try and do is when I eventually start isolating some of the things I notice help some of the groups, I want to be able to recognize quickly that I can use that in another group for a different situation. So, that is my goal hopefully by just having these focus groups. . . .
>
> MTE What happens is the tools are going to start getting ingrained in your brain and it's going to carry over to the other groups.

Even though Arden had her focus groups, she still needed to attend to other groups. Arden was able to extend what she learned with the focus groups to other groups. In Quarter 2, Arden explained:

> I feel like I'm at that stage where I check [tools] every once in a while, but I'm pretty familiar with these dynamics. . . . It's almost inherent in what I'm doing, because I don't really reference them. But, I noticed I'm carrying it over to the eighth graders as well. (Debrief Meeting 14/15)

As the various aspects of the S^{3}D Approach became integrated into her practice, Arden was able to attend to more seventh-grade groups as well as groups in her other classes in a purposeful way.

Just as the process was incremental, so too was the *change in classroom norms*. A highly effective mathematical discourse community is not going to be established immediately. Arden made this point after watching a video with ideal student-to-student interactions:

> Arden [The video showed] the perfect image of what student-to-student collaboration looks like. . . . What I notice is that this has been developed over a long time. . . .
>
> MTE That is a really important point. When we show those videos to teachers, we need to let them know that this does not happen instantaneously and then we need to support them with strategies in how you move them there. (Debrief Meeting 7)

The S^{3}D Approach involves redefining engagement in the learning of mathematics. Arden explicitly articulated to her students what success means in her class: "I discussed that our success is growth. It is not about getting the best grade possible; it is about communicating and working together" (Debrief Meeting 7). An overarching norm is the establishment of a culture of respectful collaboration:

> MTE The student-to-student interactions are very important; how we foster those with the talk moves [is] very important. But, there's also this norm above it all about just speaking respectfully.
>
> Arden That's a really big point to make. What we're talking about . . . humanizing what we're doing.

Arden also talked about reinforcing with students that they should support each other: "I've been keeping up the norm that we're a team and we help each other" (Debrief Meeting 17). However, students who have not had previous experiences communicating mathematically do not understand the various aspects of productive discourse. Arden talked about such a student in Debrief Meeting 3:

> Arden There is a level of she does not know how to communicate her thinking because this is something she has done on her own for so long.
>
> MTE . . . You are being pretty explicit—these are the norms of the classroom. That's really one important piece. But, we can say we want you to do this, but they don't necessarily know how to do it. . . . That's where this whole process comes in where the way that you are interacting with them and your actions implicitly will reinforce . . . the type of interactions that you want between the students. Again, it's just going to take time.

Through the implementation of the S^{3}D Approach, students learned to explain their ideas, listen to the ideas of others, and evaluate those ideas. Arden observed change with respect to student explanations: "[I've been] going through a lot of my groups lately and finding them saying, 'I think we need to explain this. I think we need to stop and talk about it'" (Debrief Meeting 16). Arden also observed the impact of the talk moves during the whole-class discussions:

> I've also been doing that teacher move [restate in your own words]. . . . In the main session, they're starting to expect to have to reiterate what someone said or say it in their own words. . . . I've actually also been [asking]: "Can you compare what this group did to what you did?" . . . I was really impressed. [One group said]: "In comparison to group five we did this, but similar, we did that." (Debrief Meeting 9)

Both the act of restating another person's ideas in one's own words and comparing approaches requires students to critically listen to one another. In turn, students start to critique the thinking of others. During Debrief Meeting 11, Arden and the MTE discussed reframing norms around the idea of critique:

> MTE You're basically establishing this norm that critiquing doesn't have to be negative. It's how we learn. . . .
>
> Arden I tried to publicly give them that feedback of it's okay, that we make mistakes here. . . . In the beginning, especially early norm setting, it was just drilled into them that right answers mean very little to me. It's what we learned from making our mistakes. That means more.

The students were learning that critiquing the work of others and making mistakes are an integral part of the learning process.

3.3. Use of Tools to Enhance Teacher Practice

Since the S^{3}D Approach is incremental, Arden *did not use the talk moves immediately*. During Debrief Meeting 3, Arden noticed the difference between the S^{3}D Approach and professional development that has more immediate products:

> [In] PDs, you want it to be make and take, . . . you want to open a book and find a page of a whole list of talk moves that you can do. But, that is not how [the S^{3}D Approach] works . . . because I have to consider everything that is involved with the individuals. . . . We can't just spout out a bunch of talk moves that may or may not be applicable to them.

Arden further explained how the tools associated with each phase and lens of the S^{3}D Approach *helped her identify the aforementioned norms*: "As you go through the tools, there's norms we're developing or that I realize I need to develop and maintain by doing these tools" (Debrief Meeting 5).

The tools also *reveal the intricacies of and influences upon the development of small-group-student-to-student discourse*. The tools moved Arden from intuitions about a problem of practice to understanding how to identify and address the various aspects of talk in small groups.

> There's this overwhelming feeling that I've had that [the work on small-group discourse] needs to go further, but I don't know where or how. And the tools guide me along the way exactly how [the conversations] need to go further, . . . how I can explicitly and implicitly support these conversations. (Debrief Meeting 5)

The tools were critical to Arden's engagement with the S^{3}D Approach. They allowed her to see the nature of the small-group discourse as well as how to bring about change.

In fact, as Arden progressed through the phases and lenses, each tool played a role in the development of her practice. Arden used the Group Dynamics tool to assess the dynamics of each group in her class and choose a focus group for the quarter.

> The first [class], I identified a group that I think really needs the extra support and really needs this attention. And, I even have here on my Group Dynamics tool that there was a sense of Learned Helplessness. Two of the three were Non-participatory. All of them, when they came across an Obstacle, they just sat there,

> and they would sit with their microphones off during that obstacle until there was Teacher as Authority situation where I did not just give them the answer. . . . So, that's four check marks for just this one group. (Debrief Meeting 3)

The Discourse Quality tool helped Arden identify the types of student-to-student interactions, their quality, and areas for growth. The MTE shared her observations with the Discourse Quality tool for both of the focus groups.

> MTE Both of the groups this week were not falling in that first set of dynamics where people were not participating. . . . There was full on participation, talk the majority of the time that I was in there. Once you start using the [Discourse Quality] tool . . . it's not just is this question-response occurring, it's also what's the nature of it and is it high quality? Because you can have . . . a lot of talking going on which is . . . a good first step. But, the cool part is this is showing all the areas where there can be improvement.
>
> Arden It is like I am laying it all out there. These are my flaws let's fix them.
>
> MTE But, it is also not personal. They are not your flaws The view of it is: This is where my students are currently in their discourse quality, this is giving me a baseline and allowing me to establish some goals for growth. (Debrief Meeting 5)

By using the Teacher Support tool, Arden was able to interrogate how she interacted with the groups and the implications of using Process Help rather than Product Help.

> I find myself, as I am tracking, shying away from giving the Product Help. . . . [The Teacher Support tool] is really revealing. . . . The reflective part of teaching is right there in that tool. . . . How much am I holding my scholars' hands? How much of this is their words versus my words? And that is the part that I really don't want. It should never be my words; it should always be theirs. (Debrief Meeting 8)

Following Phase 1 and the three associated tools, Arden used the Talk Moves tool to identify Process Help talk moves to use with her focus groups. After an extensive conversation during which Arden reviewed the predominant dynamics of each of her focus groups and identified which aligned talk moves she was going to use, the MTE summarized how she would document their ideas: "I'll do exactly what you said. I'll take the Talk Moves tool and jot down the notes about what we said, what the purposes [of the talk moves] are. . . . Then, there is less of a cognitive load on you . . . planning them is helpful" (Debrief Meeting 8). By using the Talk Moves tool as a planning document, Arden did not have to identify talk moves in the moment and was better prepared to implement preplanned talk moves in response to the group dynamics.

As Arden engaged with the tools, she also problem-solved how to use them efficiently. For example, Arden noticed how she used the Discourse Quality tool in a different way than the MTE:

> I was just thinking that I noticed how you use your Discourse Quality [tool], and I use it differently, where I would jot down a brief note inside the square. . . . I noticed that there's obviously a bunch of question-response pairs in one observation. But, I was only documenting one because I only had time to type out notes for that one. So, I'm going to . . . go back and jot down some notes after I've done the tick marks. (Debrief Meeting 10)

Arden actively considered how to *effectively integrate the tools into her daily practice*. In another instance, when using the Discourse Quality tool, Arden discovered a student-to-student interaction not represented in the tool.

MTE Probably 50% of those [QR1] are low-level like: What page are we on? . . . But the other 50% are [interactions] like: Dittika, do you want to plot the point? So, it's still that . . . practical question. But, it's totally different. It's inviting people to participate or orienting each other to the work. . . . It's a more important type of question.

Arden If you look at my [Discourse Quality tool from] today, I was thinking the same thing. And I said the word: Olive invited Dittika into the conversation. . . . In some way, I almost feel that's a lower tier [i.e., higher cognitive demand in Discourse Quality tool] for them to have to invite each other into the conversation, especially without prompting.

MTE . . . These [tools are] a working document. So, . . . if you're noticing other question-response pairs, if you're noticing other dynamics, add them to your toolbox. (Debrief Meeting 12)

Similarly, Arden's observations of group dynamics led to an expansion of the interpretation of the Obstacle dynamic: "[The obstacle] doesn't prevent the work, it's preventing them in how they think about [the mathematics] . . . They could communicate incorrectly" (Debrief Meeting 14/15). Overall, the tools not only served as a guide through the S^3D Approach, they also allowed Arden to take ownership of the process with respect to the logistics of employing the tools, use of the tools to set goals and make plans, and the autonomy to *expand upon the tools*.

3.4. Teacher Challenges and Incremental Change over Time

Even though the S^3D Approach is broken down into incremental steps, Arden initially experienced *challenges incorporating each phase/lens into her daily practice*. She described her experience using the Discourse Quality tool: "[Using the Discourse Quality tool] was not pretty. . . . So, that is the part that's difficult, just needing to do it and try and think about it while you are teaching. It is a lot, but you end up doing it" (Debrief Meeting 5). Due to the novelty of the tools, Arden viewed them as an additional responsibility on top of her typical teaching responsibilities rather than an integral part of her practice. For each new phase/lens, the MTE provided information and advice about how to use the tool. In Debrief Meeting 6, Arden was progressing to the Teacher Support lens/tool:

MTE Just interact with the kids intuitively and then think back: What were my natural inclinations? Where are you falling [Product Help or Process Help]? You could be all one, you could be all the other, you could be a mix.

Arden My hypothesis is I probably do both. I try to lead with Process Help and then ends up becoming Product Help. That's my hunch.

MTE Just like with Discourse Quality, it's letting us look at [the small-group discourse] more deeply. So, when we move on to Phase 2, . . . we know what we are working with, what are the goals for ourself, and our goals for our students.

The tools enabled targeted observations culminating in a baseline of the three lenses from which goals for growth were established for Phase 2.

Phase 2 introduced additional challenges, since Arden had to learn to navigate the Talk Moves tool as well as assess the impact of the talk moves on the small-group discourse (using the three tools from Phase 1). The MTE shared ideas to address this challenge of *managing multiple tools at once*: "In Phase 2, you don't necessarily need [the Teacher Support tool] because you're documenting that with your Talk Moves tool. . . . When you're in the Talk Moves tool, there's . . . a focus on Process [Help]" (Debrief Meeting 10). The Talk Moves tool not only reinforced Process Help interactions but was also organized according to the 10 dynamics in the Group Dynamics tool. Later in the same debrief meeting, the MTE continued: "It's overwhelming to have all the tools. What tool should I focus on? In

Phase 2, I would say the Discourse Quality tool. . . . The goal is that we want the quality of their discourse to improve. That's the point of the talk moves" (Debrief Meeting 10). Both the Talk Moves and Discourse Quality tools incorporated the Group Dynamics and Teacher Support Tools. Focusing on the former two tools helped Arden manage Phase 2.

Arden progressed through Phases 1 and 2 collecting information on her two focus groups. The S^3D Approach allowed her to incrementally change her practice; however, steps to make integrating the process more feasible simultaneously introduced two additional challenges. First, Arden still needed to conduct her lessons and interact with multiple small groups while slowly learning how to enhance the small-group discourse.

> MTE Even though you are [in] Phase 1, you are still doing the interactions. So, even though we will formally address the interactions down the road, right now it doesn't mean you don't think about them.
>
> Arden I do them, the ones that I can think about on the spot (What do you think about this? Can you explain in your own words what this group just said?), just the basic teacher moves that we all inherently do. That's all that I can do in those moments.
>
> MTE . . . You are still going on with your class and practices, but we are inserting some of these purposeful data collection pieces as we go through [the process]. (Debrief Meeting 3)

Second, as Arden learned more about the process, she was able to see what she learned (e.g., identification of group dynamics) in groups other than her two focus groups. She was then overwhelmed with *thinking about multiple groups*:

> Arden What makes me so fearful is that I feel where do I go from here. I see what's happening now. What's worse is I am seeing it happen in other groups where there are still people not communicating at all—so I feel that I am juggling all of these dynamics, and it is so challenging.
>
> MTE . . . That is what happens. There is this: "Wait, I have five, six groups, you are telling me to focus on two?" . . . The tools are going to start getting ingrained in your brain. It is going to carry over to the other groups. (Debrief Meeting 5)

Ironically, the strategies to simplify the process (e.g., choosing focus groups, learning one phase/lens at a time) at times made the process overwhelming. As Arden gained a deeper understanding of the intricacies of small-group discourse, she began observing multiple aspects to address across all groups.

Arden felt responsible for bringing about change in the small-group discourse and was nervous that this change would not happen. For instance, when she initially used the Discourse Quality tool, many of the student-to-student interactions were of lower cognitive demand:

> They are asking practical questions about what they need to be doing next to answer the question. . . . It is really . . . humbling me. I really want them to have deeper discussions . . .. I feel . . . almost fearful that I won't be able to get them to a place where they are having more discussion [beyond] just answering the question. (Debrief Meeting 5)

Arden also felt *frustration* with particular students, who were repeatedly not showing any improvement: "I feel so lost on what to do with them [two non-participatory students]. . . . I just don't know what else to do. . . . I know what you're saying and I am going to use that [talk move]. I guess I feel so defeated by them" (Debrief Meeting 7).

Being embedded in the process and managing all of her teacher roles and responsibilities, Arden expressed *insecurity* at times. Early in the process, Arden was *not able to see improvement* in the small-group discourse. She felt frustrated and *impatient*: "[It] feels frustrating. . . . I feel like I am not making a difference because I know their dynamics

haven't changed at all. And I don't know what to do about that" (Debrief Meeting 9). The MTE was able to see the incremental growth and used these examples to encourage Arden. For instance, in one of Arden's focus groups, Erin was a Non-Participatory Student. The Group Dynamics tool was used to show both the identification and improvement in the dynamics. The MTE wrote:

> Erin is the non-participatory student. However, the group has demonstrated progress in this area. As noted in the Discourse Quality Tool, Lila regularly checked in with Erin to see if she was keeping up with the group. Additionally, Erin asked logistical questions about completing the task to the group. (Observation 9, Group Dynamics tool)

The MTE also showed Arden the observation data in the Discourse Quality tool (Figure 2): "This is really powerful. What I was seeing before was so much QR Pair 1 and maybe a sporadic [tally below] and now they're even moving down here [QR Pairs 3 through 7]" (Debrief Meeting 9). Figure 2 shows a greater array of higher cognitive demand student-to-student interactions in both of the focus groups.

At the end of Cycle 1, Arden started to *notice improvement*: "I've spent a little bit more time in the other groups who still have non-participatory dynamics, and I think there's been a bit of improvement" (Debrief Meeting 10) and "I felt like their discourse quality has improved" (Debrief Meeting 10). In Cycle 2, Arden observed and articulated growth. For example, Arden described the student-to-student interactions between Maria and Olive during a discussion about similar figures:

> I was actually really excited. . . . They were down in [the QR pairs of higher cognitive demand]; they were asking some good questions. Olive was criticizing whatever Maria was documenting. . . . Olive would say, "Well, I disagree because it doesn't mean that they have the same scale factor. It just means that all of the sides changed by the same number." . . . And then Maria would ask her to explain that. (Debrief Meeting 12)

Not only was there improvement in the group dynamics and discourse quality early in Cycle 2 with the new groups, but Arden was also able to recognize the change herself and provide details about the nature of the improvement (guided by the tools). Further, in the latter debrief sessions of the second quarter, the conversations between the MTE and Arden predominantly focused on details of the small-group discourse and the implementation of the talk moves.

MTE The place where there can be some improvement . . . [is] these three QR fives were all Blindly Accept. Lillie said . . ., "I think distance is the independent variable." Selma [agreed]. And then Lillie said, "I think time is the independent variable," and Selma [agreed].

Arden At least Selma is agreeing.

MTE Yes, . . . normally that's a soliloquy by Lillie reasoning through [the task]. So, the positive side is that you're getting Selma inserted, Selma inserted.

Arden That is a major improvement.

MTE In the talk moves tool, [we planned] when Lillie is having a soliloquy, insert yourself at that point and ask the other students to respond to the question.

Arden I did it. I did it.

MTE . . . That's exactly what that they needed. You [said], "I'm stopping you, Lillie. You two [Selma and Milton], I want you to respond to this. (Debrief Meeting 18)

Arden saw the impact of the talk moves on incrementally enhancing the student-to-student talk. Becoming more familiar with the S^3D Approach resulted in the debrief sessions

transitioning from conversations about the tools to conversations about what she was learning from the tools.

Question	Response	Focus Group 1	Focus Group 2
1. *A* asks *B* a practical question	*B* answers *A*'s question	*III*	*IIIIII*
2. *A* asks *B* a question about previously learned content	*B* answers *A*'s question	*I*	
3. *A* asks *B* to show work	*B* shows own work	*II*	
4. *A* asks *B* to explain work	*B* explains own work	*III*	*II*
5. *A* asks *B* to evaluate work	*B* evaluates *A*'s work	*I*	*I*
6. *A* criticizes *B*'s work	*B* justifies own work	*II (non-math) I*	*III*
7. *A* rejects *B*'s justification	*B* reconstructs own work		*I*
8. *A* suggests a strategy to the group	The group tries the strategy		

Figure 2. The Discourse Quality tool with data from Observation 9.

4. Discussion

Otten et al. (2022) posit that incremental PD requires "redefining success, rethinking the starting points for PD, and new mechanisms for sustainability and scaling" (p. 1447). The S^{3}D Approach itself and Arden's experiences engaging with it reflect several of these characteristics, reveal the implications of incremental change, and open up further questions for study. The S^{3}D Approach centers on changes to a teacher's regular practice (Otten et al., 2022) as long as the teacher, like Arden, regularly incorporates small-group work into instruction. Further, the S^{3}D Approach is designed to be accessible to teachers in several ways.

First, the approach has progressive *starting points*. In particular, it involves two phases, assessing the current discourse of groups and then using talk moves to enhance the discourse. The first phase is broken down into three different ways of examining small-group discourse (Group Dynamics, Discourse Quality, and Teacher Support). Arden focused on each of the lenses of the first phase one at a time. She then used this multifaceted assessment to purposefully plan Process Help talk moves that aligned with each group's dynamics to improve the discourse quality. Another component of *rethinking starting points* is the identification of focus groups. Instead of attempting the approach with all of her groups in all of her classes, Arden identified one group in each of her two seventh grade mathematics classes. Implementing the approach with just two focus groups allowed Arden to become familiar with the process on a much smaller scale.

Second, the S^{3}D Approach includes tools associated with each phase/lens (National Academy of Education, 1999). Each of the tools played a role in helping Arden understand the intricacies of small-group discourse. As Steele states, "Shifting to a more discourse-centered classroom certainly means changing our interactions with students during the lesson. Those interactions, however, are more effective when we use tools to plan for discourse" (Steele, 2019, p. 355). The Group Dynamics tool helped Arden recognize dynamics that could inhibit productive mathematical discourse (e.g., Non-participatory Student) and then use this information to identify focus groups. With the Discourse Quality tool, Arden was able

to categorize and assess the quality of student-to-student interactions within groups and, based on this evaluation, set goals to move the students to interactions of higher cognitive demand. The Teacher Support tool provided a framework to distinguish between her teacher moves that fostered student-to-student talk and those that did not. The Talk Moves tool served as a planning document for the Process Help talk moves that Arden would use to address group dynamics and enhance discourse within the small groups. "Identifying and naming these moves, and then carefully planning to use them, supports teachers in thinking more strategically about mathematics classroom discourse" (Cirillo et al., 2014, p. 141). The tools also helped Arden develop general norms around collaboration, including speaking respectfully, helping peers, critically listening to each other, and ensuring that all members understand the work of the group (Manouchehri & Enderson, 1999). Additionally, Arden established sociomathematical norms, specifically "what counts as an acceptable mathematical explanation and justification" (Yackel & Cobb, 1996, p. 461). Overall, the tools had multiple roles. They provided information about different aspects of small-group discourse and associated norms, served as a means of documentation during lessons, and enabled more formalized reflection as well as guided conversations about and evaluations of the small-group discourse in Arden's classes (Cirillo et al., 2014).

Third, the S^3D Approach allowed Arden a level of autonomy, or a feeling of ownership of the process (Power & Goodnough, 2019). Arden made decisions about at what pace she moved through the lenses and phases. For instance, in the first cycle, she spent more time with the Phase 1 lenses. In comparison, in the second cycle, she felt more familiar with the tools and chose to move into Phase 2 more quickly. By using the tools and reflecting on the information about the small-group discourse, Arden was able establish goals for growth. In particular, the Discourse Quality tool indicated directions for higher quality student-to-student talk and, as previously mentioned, the Talk Moves tool provided guidance with respect to talk moves that would help students move in that direction. Arden was also reminded to consider each of the tools as a working document. In fact, she identified a student-to-student interaction (inviting peers to participate) that was not present in the Discourse Quality tool. She also expanded the meaning of some of the Group Dynamics and identified different ways to frame the Talk Moves. Regardless of the S^3D Approach being incremental, incorporating the tools into Arden's practice was difficult, especially early in the process. In response to this challenge, Arden considered ways to more effectively and efficiently employ the tools. In fact, she was instrumental in transitioning the tools from a paper-and-pencil format (Appendix A) to an intermediary pilot of an electronic format to the formal development of an app. Throughout the PD, Arden was actively involved in making logistical and data-based pedagogical decisions.

Fourth, the S^3D Approach involves repeated cycles of going through Phase 1 and then Phase 2. Each quarter, Arden created new groups, identified focus groups, progressed through the three lenses of Phase 1, and then planned and implemented talk moves in Phase 2. The present study focuses on the first two cycles. The process was quicker moving from the first to the second cycle. Arden spent the majority of the first quarter in Phase 1. In comparison, she spent the majority of the second quarter in Phase 2. In the second cycle, Arden was more familiar with the various lenses of Phase 1 and the use of the associated tools. Therefore, she was able to move quickly through Phase 1 and into Phase 2. The process and tools became integrated into her practice, evidence of the *sustainability* of the approach. Otten et al. (2022) similarly described how incremental changes "can carry forward for the teacher by becoming their new instructional 'habits'" (p. 1447).

Arden's increased familiarity with the process changed the nature of the conversations during the debrief sessions. Initial discussions centered on learning the details of the phases and lenses and the structures of and strategies for utilizing the tools. Arden also shared

various challenges that she was experiencing, such as using the tools in addition to all of her other responsibilities as a teacher. Even though Arden used the tools with just two focus groups, she struggled with managing all of her groups. The incremental approach contributed to this challenge and, in the long term, its resolution. At first, Arden interacted with all of her groups and knew that they needed her support. However, she felt that, with her developing understanding, she was not able to provide the necessary and appropriate attention. Over time, as her understanding of the process developed and became part of her *instructional habits*, she was able to implement the approach with groups other than the focus groups, further supporting the *sustainability* as well as the *scaling* of the process.

During the early conversations, Arden also expressed impatience and insecurity. She wanted to see greater improvement and, when she did not, she was unsure of her implementation of the approach. In fact, on two occasions, she asked MTE, "Did I do something wrong?" and "Maybe I'm not doing something right?" (Debrief Meetings 9 and 13). Further, she sometimes felt that the tools were revealing her flaws. Through engaging in incremental PD, Arden needed to *redefine success*. She needed to be reminded that with persistence, her practice and the small-group discourse would change over time. Rather than exposing faults, the tools served as a baseline for growth and a means of seeing minor incremental change, especially with students that posed the greatest challenges.

Since Arden was engrossed in learning and enacting the approach as well as implementing her lessons, she was initially not able to see improvement. The MTE, who was observing focus groups for an entire class period with that as her only focus, did see growth in the Group Dynamics and enhancement in the Discourse Quality. The MTE shared this information with Arden, providing positive reinforcement of her work and ways to recognize the incremental change. Toward the end of the first cycle, Arden started to recognize and share improvement on her own. In the second cycle, the substance of the discussions became more focused on the intricacies of the small-group discourse based on Arden's observations and the complexities of the strategies to bring about incremental change. These findings support the hypothesis of Otten et al. (2022): "It seems plausible that when teachers enact small improvements, they may gain confidence and feelings of success, which may lead them towards the addition of more small improvements" (p. 1447). Even with the change in groups (and the introduction of new group dynamics) in the second quarter, improvement from the first quarter was *sustained*, allowing Arden to build upon the progress in the second quarter. In other words, the growth was cumulative.

The MTE played a critical role in providing guidance about the S^3D Approach, conducting weekly observations of the focus groups, and debriefing with Arden about her experiences each week. Further, the MTE encouraged Arden to persist through the incremental process, especially when she felt impatient, frustrated, and insecure. With this perseverance, Arden was eventually able to realize improvement in the small-group discourse. A major limitation of the current iteration of the S^3D Approach is that it was conducted on a small scale. Just one teacher worked one-on-one with the MTE, therefore raising questions about the *scaling* of the process. The S^3D Approach could be conducted with a larger group of teachers either led by an MTE or independent of an MTE. In the first case, MTEs (including university mathematics teacher educators, mathematics coaches, or department chairs) could facilitate the incremental PD (Kohen & Borko, 2022). Quebec Fuentes (2020) designed a Professional Development Guide for structuring and implementing a sequence of sessions. In the latter case, teachers could collaboratively engage with the process, observe each other's classes (Chen & Chan, 2022), and serve as a support through challenges. During Debrief Meeting 5, Arden explained the importance of working with other teachers: "It's really helpful when using the tools to have someone to collaborate

with on it. So, maybe as another piece of advice to someone who would be using [the S^{3}D Approach] is find someone else in the school who would be willing to also do this."

The infrastructure exists for the approach to be scaled up whether led by an MTE or not. A variety of resources that meet the needs and capacities of different teachers exist. For example, Quebec Fuentes (2020) designed a PD resource, including a breakdown of the phases and lenses, examples of teachers using the approach, and prompts and worksheets to support teachers as they engage in and reflect upon the approach. However, this may be overwhelming for teachers. As an alternative, Quebec Fuentes has created a series of social media posts across various outlets (Instagram [s3dmathed], X [@s3d_math], and TikTok [@s3dmathed]). The posts include information and videos about each of the incremental aspects of the S^{3}D Approach and their implementation with the tools. The tools are available in both paper-and-pencil format (linktr.ee/s3dmathed) or embedded within an app. "The freedom to deliver specific, actionable practices via on-demand formats, such as videos, downloadable documents, or social media suggestions will allow for the scaling of those efforts by the PD creators and practitioners themselves" (Otten et al., 2022, p. 1447).

Several questions remain and can serve as the focus of future research. First, how can the S^{3}D Approach be exposed to a greater number of practitioners? In other words, with the aforementioned mechanisms in place, how does one support the expansion of the reach of the approach? Second, how is the approach being implemented independent of the MTE? Future studies can examine how communities of practice with the shared goals of fostering small-group discourse (Quebec Fuentes & Spice, 2017) work with the incremental PD.

5. Conclusions

Bishop (2021) described the implications of her study that examined the relationship between teachers' attending to student thinking and learning outcomes:

> Analyses indicate that relatively small increases of highly responsive teacher moves may have significant effects on achievement. In terms of instructional implications, these kinds of smaller, incremental changes may be more realistic for teachers, teacher educators, and researchers who hope to increase the responsiveness of classroom discourse—changing a small number of turns of talk from low to high levels of responsiveness around a preplanned task or concept might be a productive starting point for shifting classroom discourse. (p. 502)

The S^{3}D Approach is purposefully designed to support incremental change in a teacher's practice to foster small-group, student-to-student discourse. The current study demonstrates how, through a reframing of starting points and the definition of success, incremental PD supports sustainable changes in a teacher's practice and student engagement in productive mathematical discourse.

Funding: The creation of the app was funded by Andrews Institute for Research in Mathematics & Science Education (04062023). Publication charges for this article were supported by the TCU Library Open Access Fund.

Institutional Review Board Statement: The study was conducted in accordance with the Declaration of Helsinki and approved by the Institutional Review Board (or Ethics Committee) of Texas Christian University (protocol code IRB#2022-200, 9 July 2022).

Informed Consent Statement: Informed consent was waived due to the study being deemed exempt by the IRB.

Data Availability Statement: The data presented in this study are available on request from the corresponding author due to privacy restrictions.

Acknowledgments: I would like to express my gratitude to the teacher who participated in the current iteration of the S^3D Approach.

Conflicts of Interest: The author declares no conflicts of interest. The funder had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Appendix A

	Dynamic										
	Participation				Momentum			Definition of Success			
Group	**Learned Helplessness** The group members need you, or a student that typically leads the group, to initiate work on a task. There is either no communication or off-task conversations.	**Help-Leave-Silence** You initiate communication about a task between group members. The conversation terminates after you depart.	**Own Zones** The group members individually work on a task.	**Non-participatory Student** One or two group members are not partaking in the conversation about a task between the other group members.	**Obstacle** The group members have worked on a task and ask you for help. To respond appropriately, you need to know what work has been done.	**Unsuccessful Help** Despite one group member trying to help another, the group member receiving assistance remains confused often not admitting to his or her remaining questions.	**Dominant Student** One group member dictates the conversation often minimizing or completely overlooking the ideas of the other group members.	**Rush to Complete Task** The group members' priority is to complete a task with little to no assessment of the approach and/or solution.	**Teacher as Authority** The group members perceive you as the sole person who can provide help.	**Blindly Accept Work of Others** One or more group members concur with erroneous, incomplete, or incomprehensible reasoning.	Notes
1											
2											
3											
4											
5											
6											
7											
8											

Figure A1. The Group Dynamics tool.

Question-response Pair		Groups							
Question	Response	Group 1 (Focus)	Group 2 (Focus)	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
1. *A* asks *B* a practical question	*B* answers *A*'s question								
2. *A* asks *B* a question about previously learned content	*B* answers *A*'s question								
3. *A* asks *B* to show work	*B* shows own work								
4. *A* asks *B* to explain work	*B* explains own work								
5. *A* asks *B* to evaluate work	*B* evaluates *A*'s work								
6. *A* criticizes *B*'s work	*B* justifies own work								
7. *A* rejects *B*'s justification	*B* reconstructs own work								
8. *A* suggests a strategy to the group	The group tries the strategy								

Figure A2. The Discourse Quality tool.

Group	Type of Help		Notes
	Product	Process	
1			
2			
3			
4			

Guiding Question	Indicators and Examples	
	Product Help	Process Help
What is the focus of teacher questions or prompts?	Mathematics Content	Helping Students Communicate with One Another about the Mathematics
What is the communication pattern?	Teacher-Student-Teacher-Student	Teacher-Student-Student-Student
How many students are participating in the conversation?	One	All
Who is leading the conversation?	Teacher	Students
Who is doing the majority of the talking and thinking?	Teacher	Students

Mr. Warren	Yes, what's wrong?
Gina	How do you know how much to take out? That always confuses me.
Helen	I'm stuck on that one, too.
Mr. Warren	Let's look at it systematically. So, we're going to look just at the coefficients. What is the largest number that divides 12 and 15?
Gina	3.
Mr. Warren	So, that's what you factor out. So, now look at the variables. What's the largest factor?
Gina	x.
Mr. Warren	Now, what's left over after you factor out $3x$?
Gina	$4x$.
Mr. Warren	Now, do the second term. Three times what gives you –15?
Gina	–5.
Mr. Warren	So, in parentheses, you have $4x - 5$. Now, repeat the process for the next two terms. What's the largest number that divides both?
Gina	2.
Mr. Warren	Now, factor out the 2.
Gina	$4x$.
Mr. Warren	And?
Gina	–5.
Mr. Warren	Yes, and notice that $4x - 5$ is the same. So, those are your two factors.
Brooke	Right here, it's negative five. So, would that make it positive five?
Mr. Warren	Did you ask Paige?
Brooke	No.
Paige	Yes. It would turn positive.
Mr. Warren	Ask Zoe as well. See what she thinks.
Zoe	I agree because –5 is negative and $-b$ is negative. A negative and negative makes a positive.
Brooke	I'll do that. Five plus or minus the square root of …
Paige	I just got confused at one point because I ended up with square root of one over four.
Brooke	Well, wait. Let me do it.
[Brooke, Paige, and Zoe continue to discuss the problem.]	

Figure A3. The Teacher Support tool.

Element/Dynamic	Description	Talk Moves	Example Talk Moves	Notes
PARTICIPATION Learned Helplessness	The group members need the teacher, or a student that typically leads the group, to initiate work on a task. There is either no communication or off-task conversations.	Clarify questions Redirect questions to group Direct explanations to group members Refer to other resources	Can you ask me a specific question? Teacher restates student question directed at teacher to the rest of the group. Explain that to your group members. Where can you find the answer to your question?	
Help-Leave-Silence	The teacher initiates communication about a task between group members. The conversation terminates after the teacher departs.	Leave group with a task Follow-up on progress	Each write down your explanation and then compare it with those of your groupmates. Did you compare your explanations? What are the similarities and differences?	
Own Zones	The group members are individually work on a task.	Redirect questions Individual work and then compare strategies	Could you answer her question? Explain your strategies to each other and then compare them.	
Non-participatory Student	One or two group members are not partaking in the conversation about a task between the other group members.	Explain what has been done Another student explains Restate in own words Answer another student's question	Can you explain what they were just discussing? Can you explain your strategy to him? To person being helped: Now what are you going to do to figure out …? Could you help her with her question?	

Element/Dynamic	Description	Teacher Interactions	Example Teacher Interactions	Notes
MOMEMNTUM Obstacle	The group members have worked on a task and ask the teacher for help. To respond appropriately, the teacher needs to know what work has been done.	Ask for explanation – Push for clarity Focus on Errors	What did you do to figure out …? Without directly identifying error, the teacher models the process of evaluating work. Do you agree with what he did? Why not? What is the difference between your strategies?	
Unsuccessful Help	Despite one group member trying to help another, the group member receiving assistance remains confused often not admitting to her remaining questions.	Restate in own words Agree with restatement	Can you explain what she is saying/doing? Do you agree with how he described your strategy/reasoning? Why or why not?	
Dominant Student	One group member dictates the conversation often minimizing or completely overlooking the ideas of the other group members.	Restate in own words Highlight overlooked idea of another student	Can you explain what they just said? How did you (ignored student) solve the problem?	
DEFINTION OF SUCCESS Rush to Complete Task	The group members' priority is to complete a task with little to no assessment of the approach and/or solution.	Compare strategies Evaluate work of others	Compare your answers/strategies. Can you determine which answers/strategies are correct and which are incorrect?	
Teacher as Authority	The group members perceive the teacher as the mathematical authority.	Redirect questions to group Ask student to redirect question to group Explain work to others Ask others to evaluate work	A student asks the teacher a question. The teacher restates the question to the group. Can you ask your question to them (other group members)? Explain to the group what you did and maybe they can come up with a suggestion. Do you agree with what he said? Why or why not?	
Blindly Accept Work of Others	One or more group members concur with erroneous reasoning.	Restate in own words Evaluate student's ideas	Can you explain it now in your own words? Does that explanation make sense? Why?	

Figure A4. The Talk Moves tool.

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